信号与系统
SIGNALS AND SYSTEMS

信息工程学院 张黎

一、教材

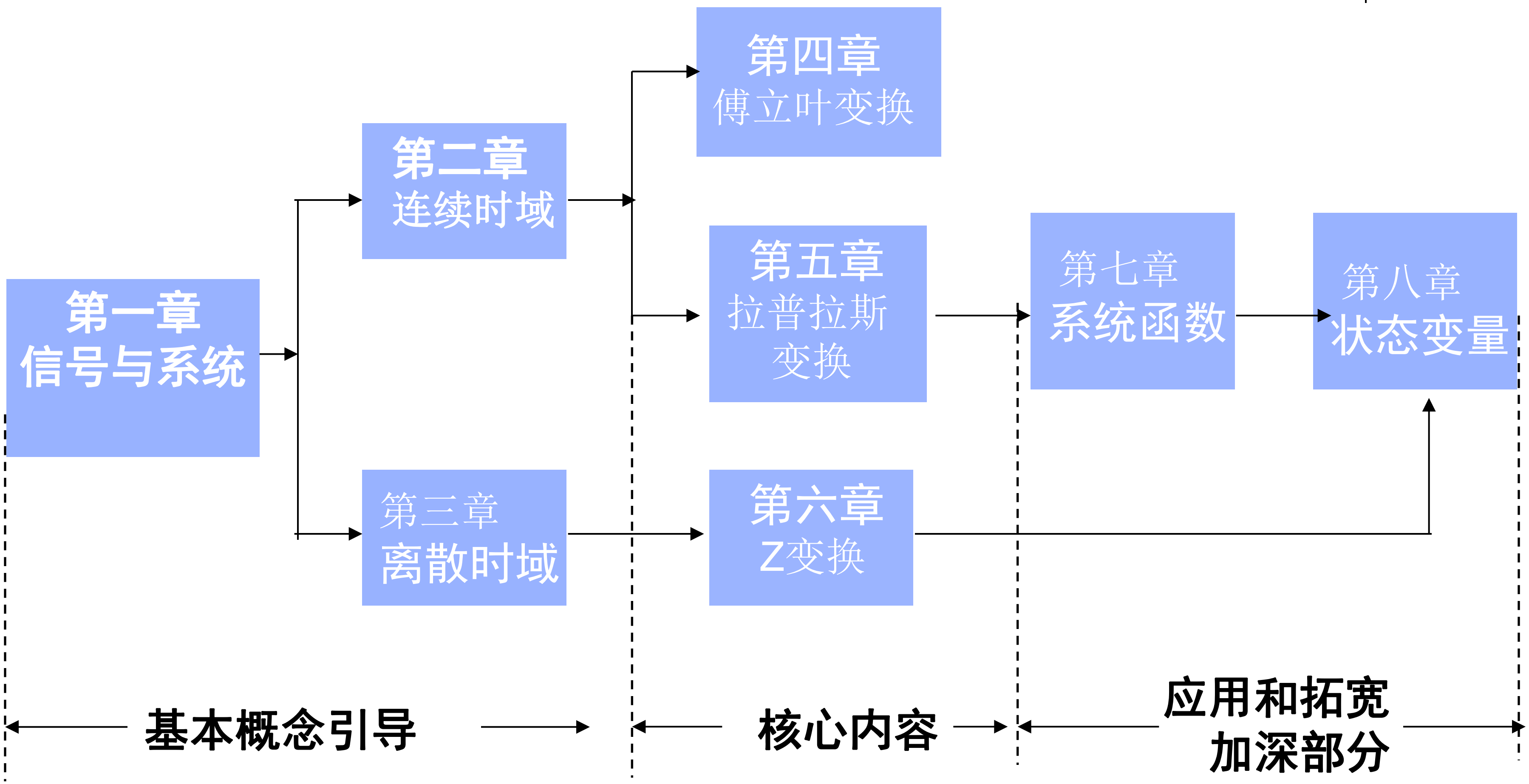
信号与线性系统分析 吴大正 主编(第四版)

二、参考书

Signals & Systems (second edition) A. V. Oppenheim

信号与系统	郑君里等	高等教育出版社
信号与系统	曾禹村等	北京理工大出版社
信号与系统分析	罗永光等	国防科大出版社

三、学习的主要内容



第一章
信号与系统

第二章
连续时域

第三章
离散时域

第四章
傅立叶变换

第五章
拉普拉斯
变换

第六章
Z变换

第七章
系统函数

第八章
状态变量

基本概念引导

核心内容

应用和拓宽
加深部分

第一章　信号与系统

要点：

1. 信号的运算（+，−，•，自变量变换）

2. 阶跃函数　$\varepsilon(t)$，冲激函数　$\delta(t)$
   定义及其性质。

3. 线性非时变系统的性质

4. 系统的描述及框图模拟

§1.1 绪言 §1.2 信号 §1.3 信号的基本运算

一.信号(Signal)　　　　　　　　　函数 (function)

传送消息的符号或载体(物理量)

二. 系统 (System)

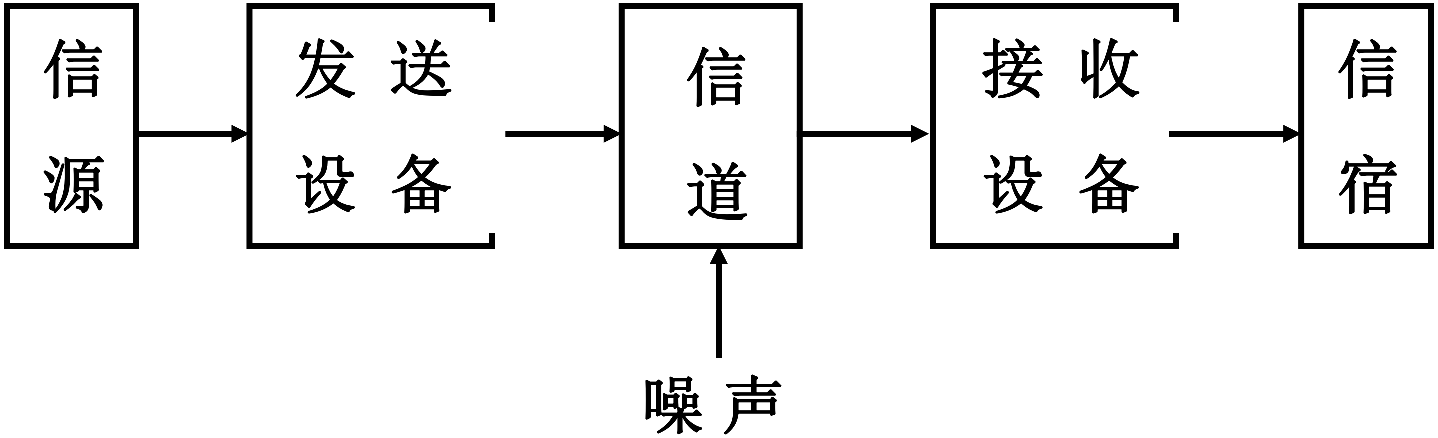
1. 若干相互联系,相互作用的事物(部件)组合而成的,具有某种特定功能的整体.

2. 信号系统：信号的产生,传输,处理,存储,转换.

3. 系统的简化框图

$$f(t) \longrightarrow y(t) \qquad y(t) = T[f(t)]$$

$$f(k) \longrightarrow y(k) \qquad y(k) = T[f(k)]$$



信源

发送设备

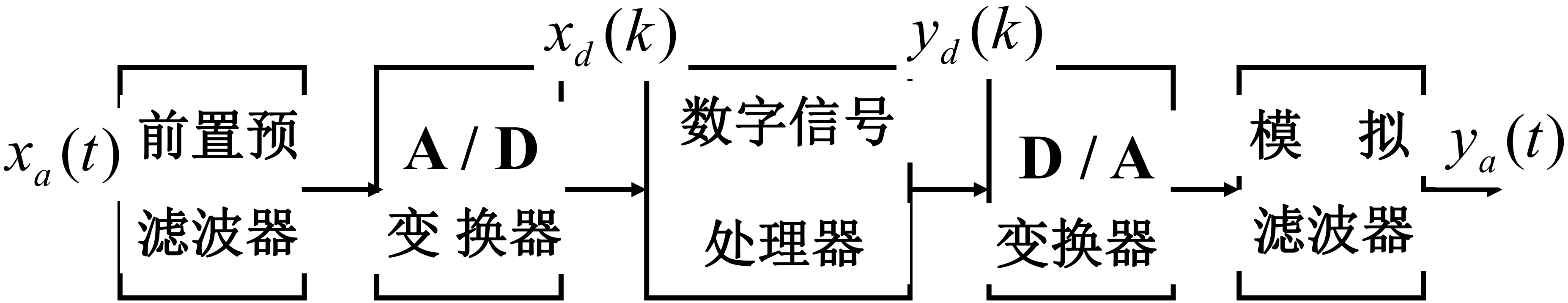
信道

接收设备

信宿

噪声

通信系统框图



$x_a(t)$

前置预滤波器

A / D 变换器

$x_d(k)$

数字信号处理器

$y_d(k)$

D / A 变换器

模拟滤波器

$y_a(t)$

音响处理系统框图

三. 信号的分类

1． 连续信号：定义域连续　$\cos(\beta t), \varepsilon(t), e^{-\alpha t}$

离散信号：定义域离散　$\cos(\beta k), \varepsilon(k), a^{k}$

模拟信号：定义域连续，值域连续

数字信号：定义域离散，值域离散

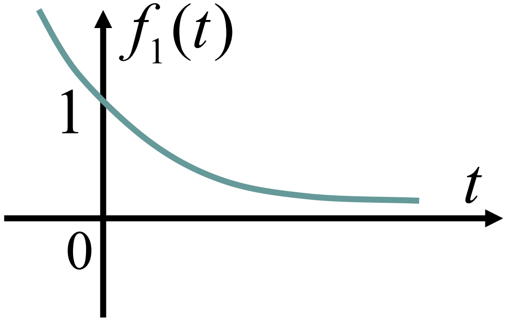
2． 实信号(实自变量t,k的实函数)

$$\cos(\beta t), \varepsilon(t), e^{-\alpha t}, \delta(t)$$

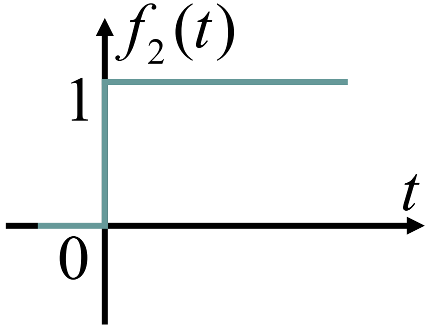
$$\cos(\beta k), \varepsilon(k), a^{k}, \delta(k)$$

复信号(实自变量t,k的复函数)

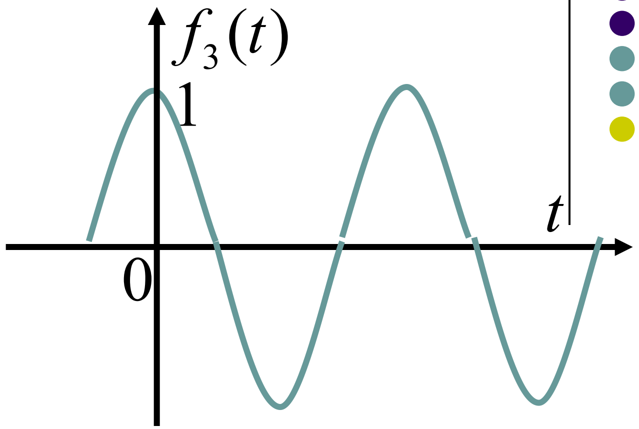
$$e^{st}, s = \sigma + j\omega, e^{j\omega t}, z^{k}, z = \rho e^{j\theta}$$



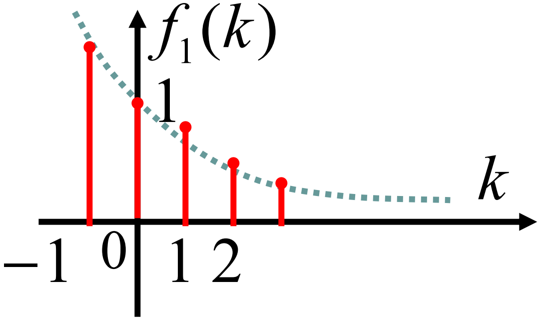
$$f_1(t) = e^{-\alpha t}$$



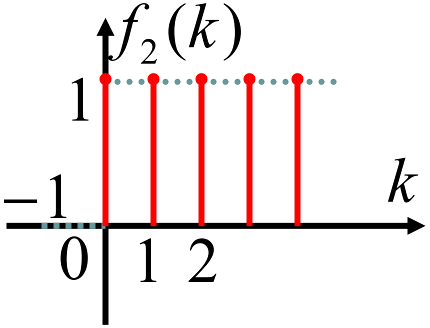
$$f_2(t) = \varepsilon(t)$$



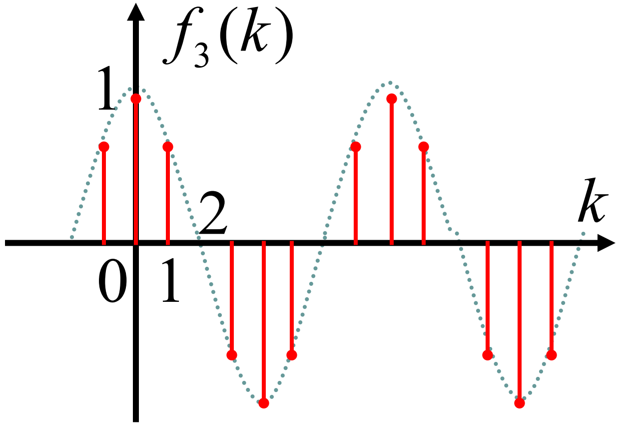
$$f_3(t) = \cos(\omega t)$$



$$f_1(k) = a^k$$



$$f_2(k) = \varepsilon(k)$$



$$f_3(k) = \cos(\beta k)$$

3. 周期信号:

$$f(t)=f(t+T),\forall t,\quad \cos(\omega t),\ldots\ldots$$

$$f(t)=f(t+mT),m=\pm1,\pm2,\ldots\ldots$$

$$f(k)=f(k+N),\forall k,\quad \cos(\beta k),\ldots$$ *

$$f(k)=f(k+mN),m=\pm1,\pm2,\ldots\ldots$$

非周期信号

4. 一维信号

多维信号

四. 信号的基本运算

1. 两个信号的加,减,乘运算

$$f_1(t)\pm,\times f_2(t),\ f_1(k)\pm,\times f_2(k)$$

2. 自变量变换 ***Transformation of the independent variable***

(1) 平移:

Time shift

$$f(t) \to f(t \pm t_d),(t_d > 0)$$

$$f(k) \to f(k \pm m),(m > 0)$$

(2) 反折:

Time reversal

$$f(t) \to f(-t)$$

$$f(k) \to f(-k)$$

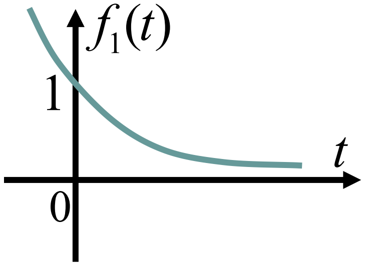
(3) 时间尺度变换:

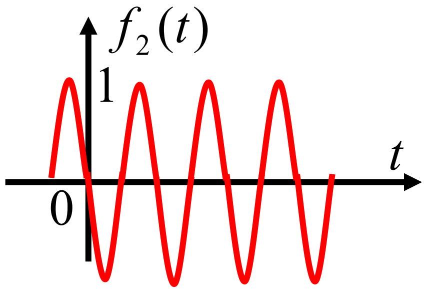
Time scaling

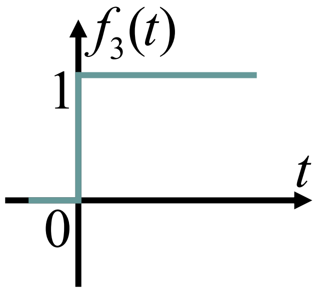
$$f(t) \to f(at) \begin{cases} a > 1 \\ 1 > a > 0 \end{cases}$$

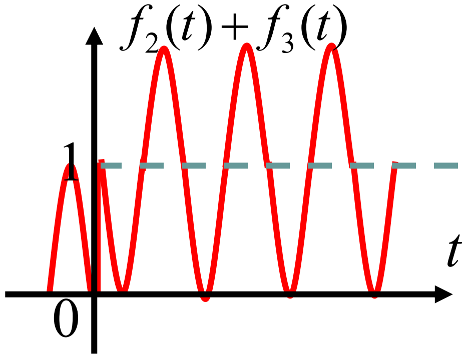
$$f(k) \to f(ak)^{*} \begin{cases} a > 1 \\ 1 > a > 0 \end{cases}$$

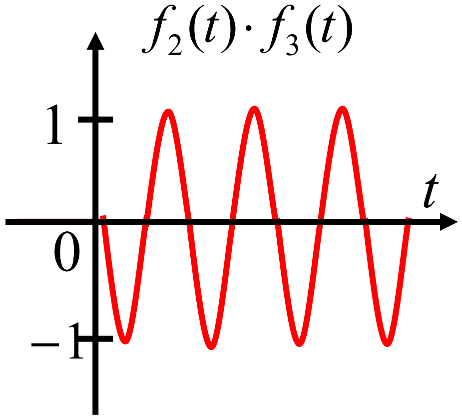
例 1.1-1

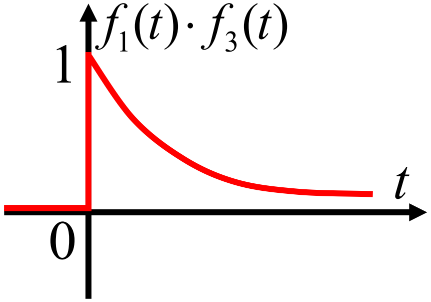

$f_1(t)$
1
0
t

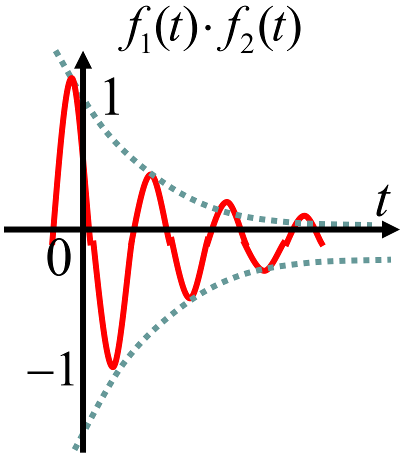

$f_2(t)$
1
0
t


$f_3(t)$
1
0
t

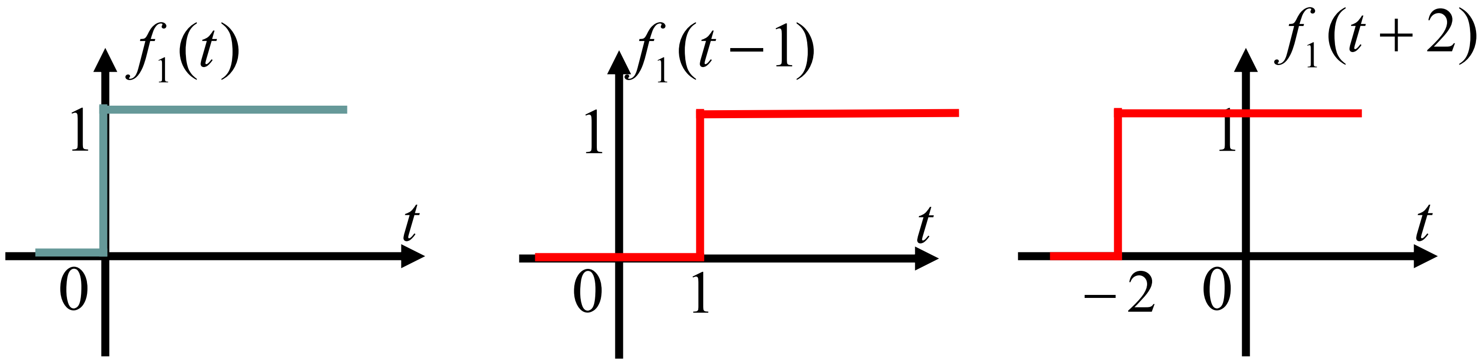

$f_2(t)+f_3(t)$
1
0
t


$f_2(t)\cdot f_3(t)$
1
0
-1
t

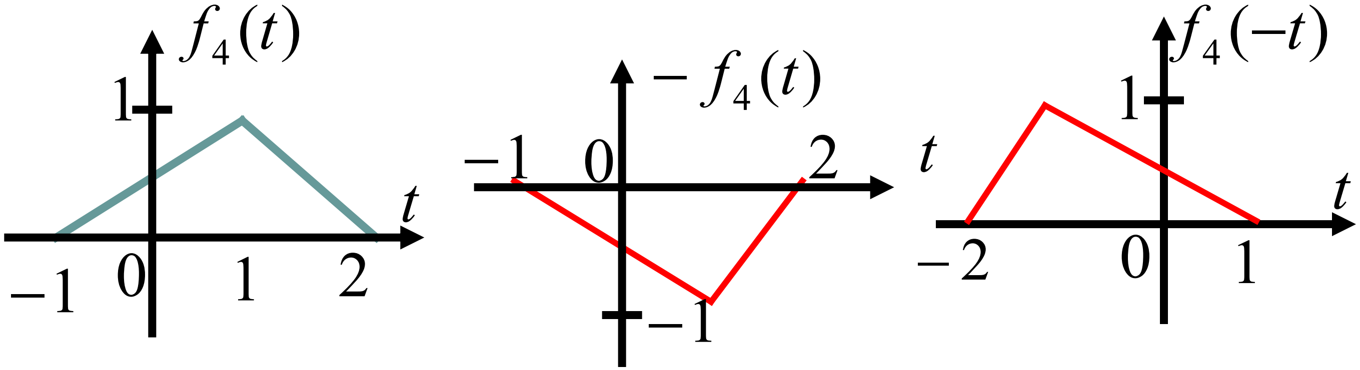

$f_1(t)\cdot f_3(t)$
1
0
t


$f_1(t)\cdot f_2(t)$
1
0
-1
t

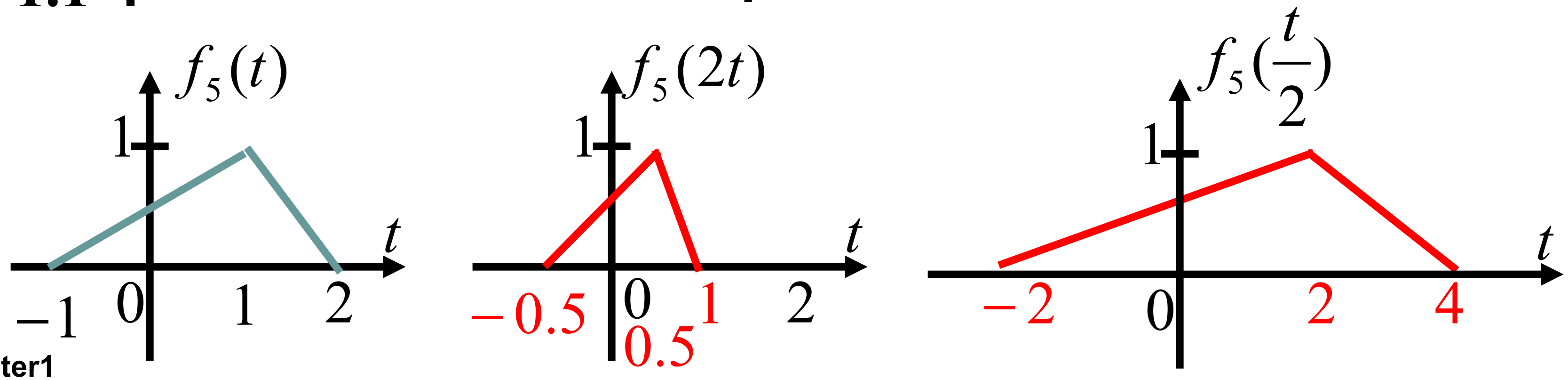
例 1.1-2



例 1.1-3

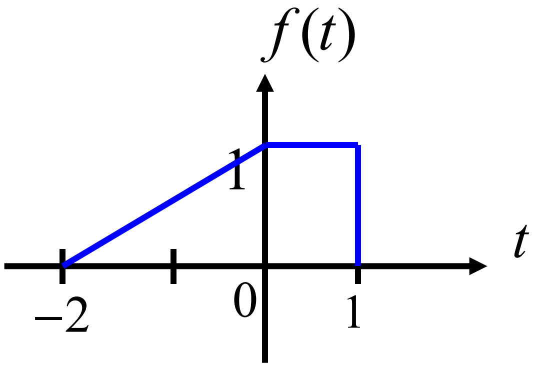


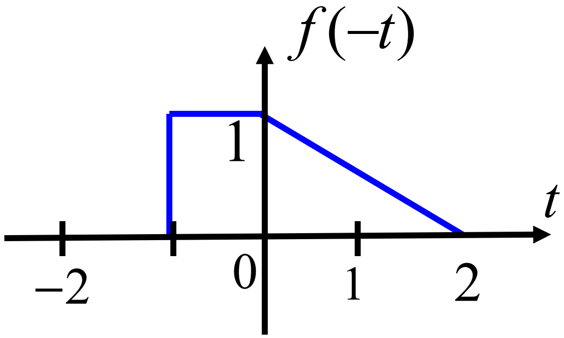
例 1.1-4

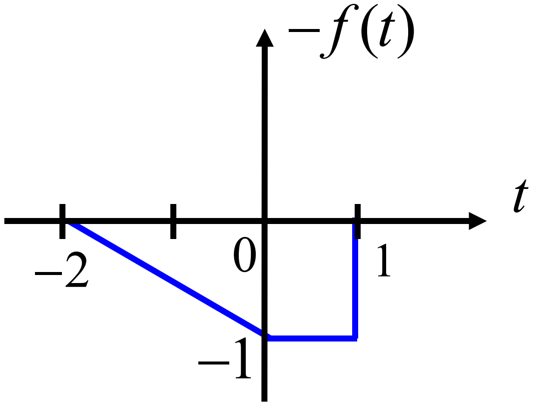


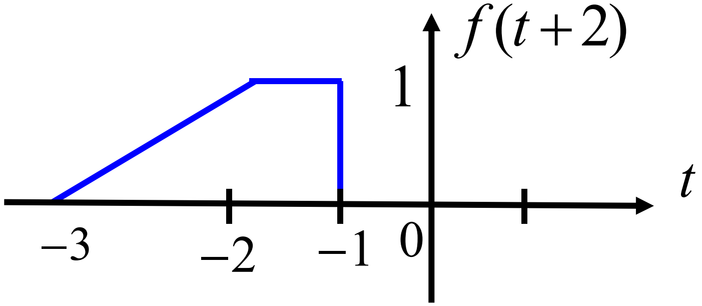
例 1.1-5　已知 $f(t)$ 波形如下，求

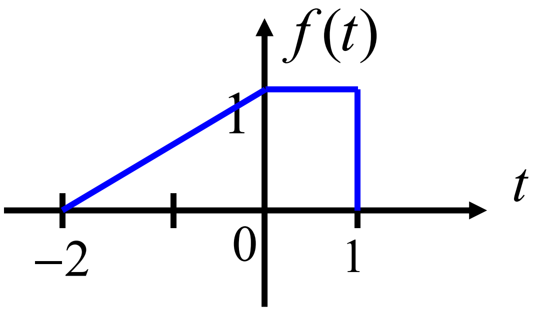
$$f(-t), -f(t), f(t+2), f(t-2), f(2t), f(\frac{1}{2}t), f(-2t+4)$$

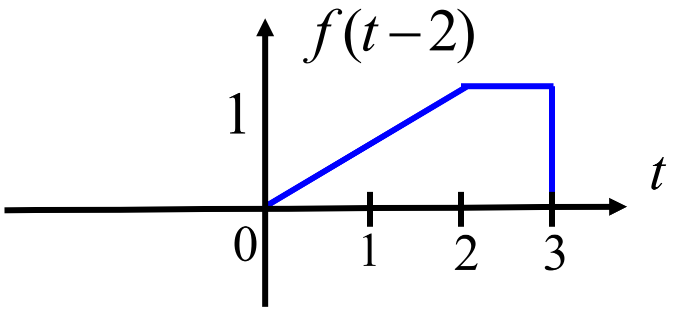


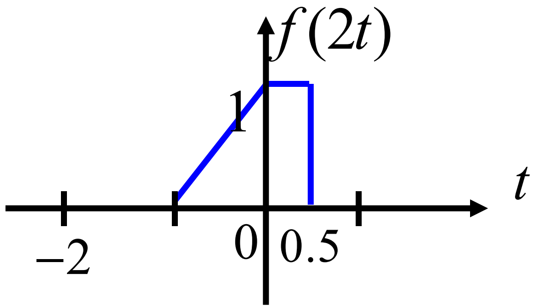


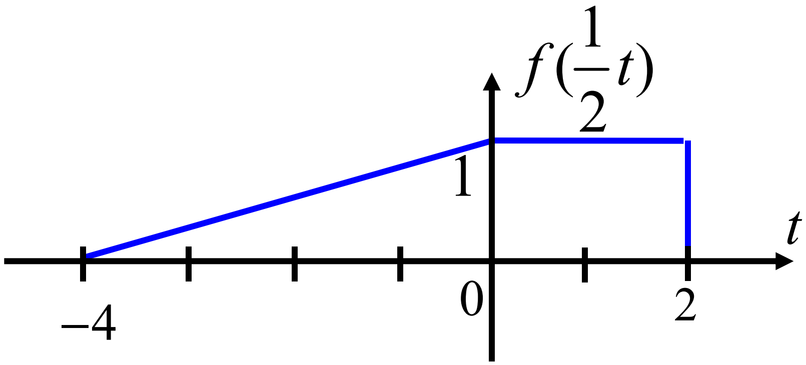




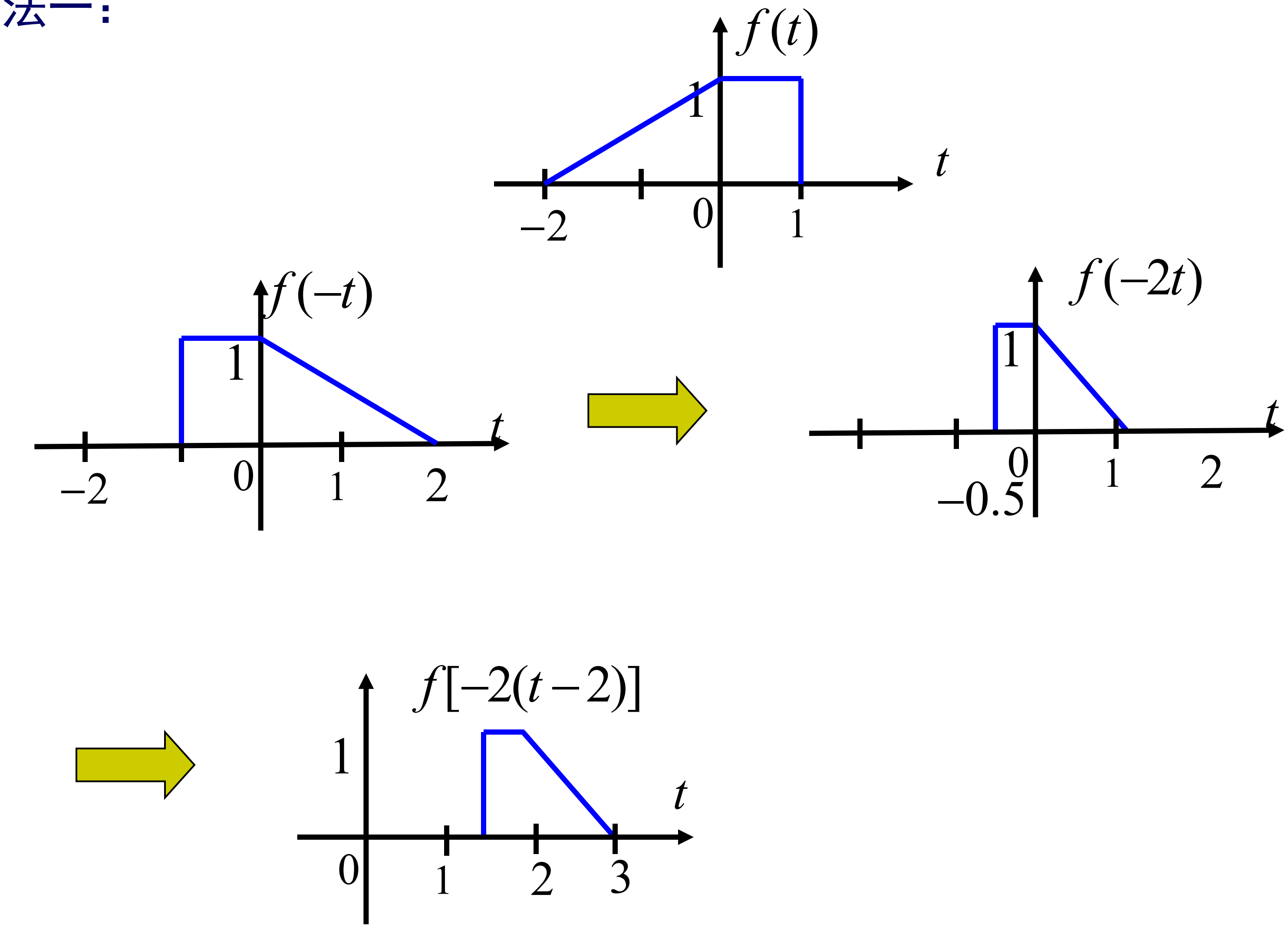

$f(t)$
1
t
-2
0
1


$f(t-2)$
1
t
0
1
2
3

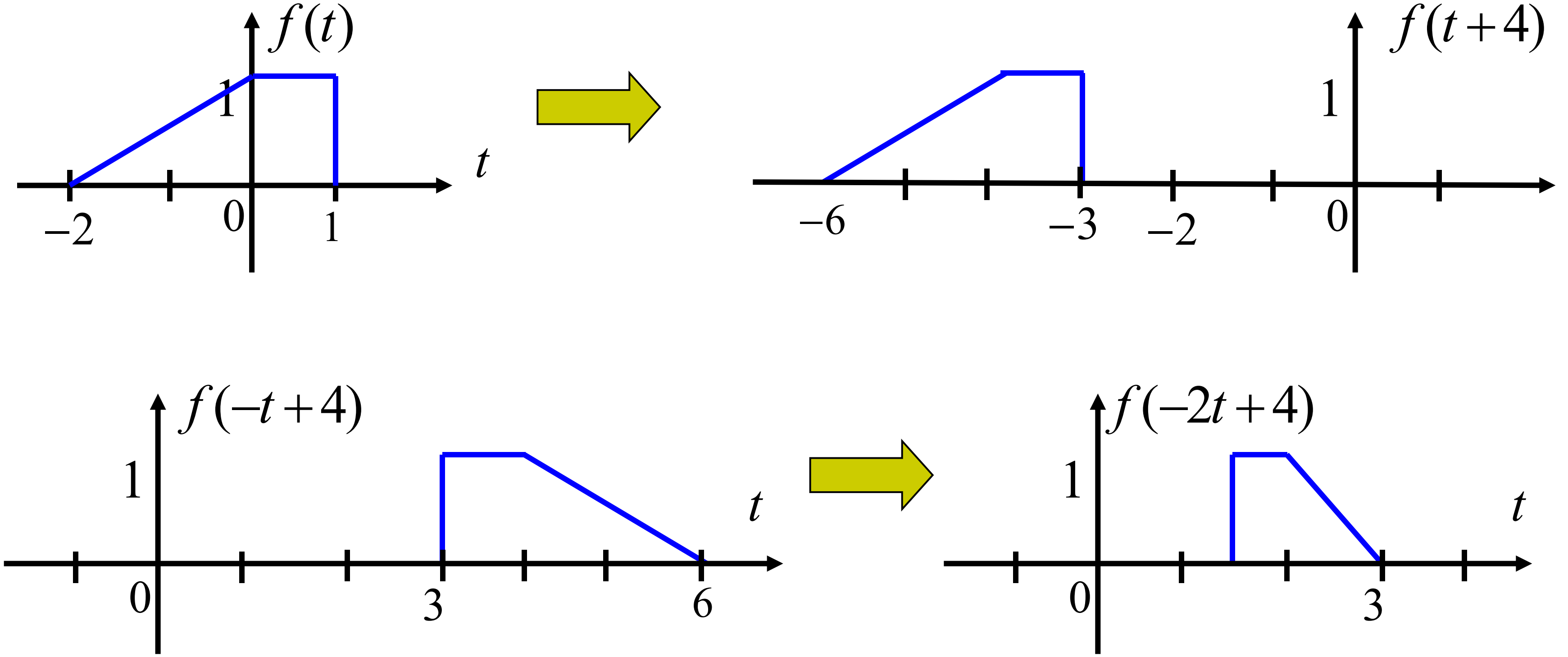

$f(2t)$
1
t
-2
0
0.5


$f(\frac{1}{2}t)$
1
t
-4
0
2

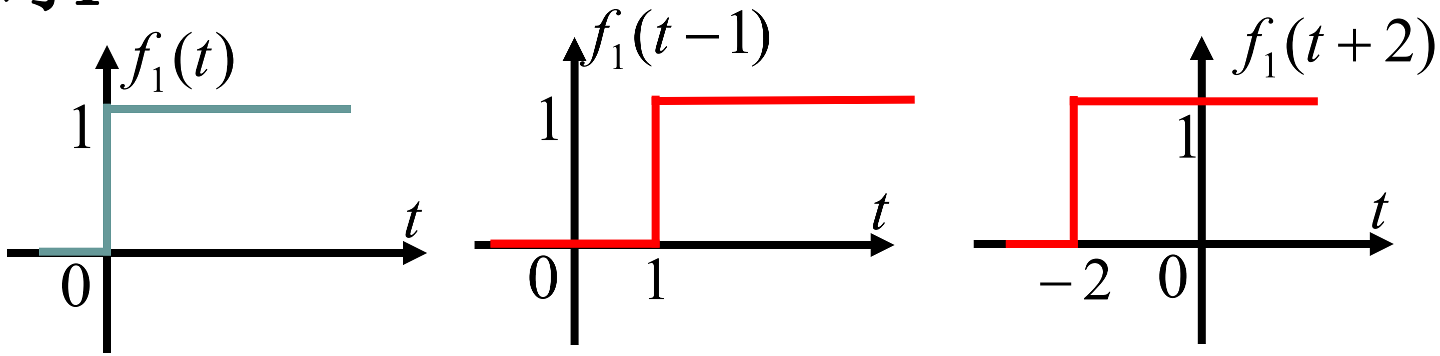
法一：



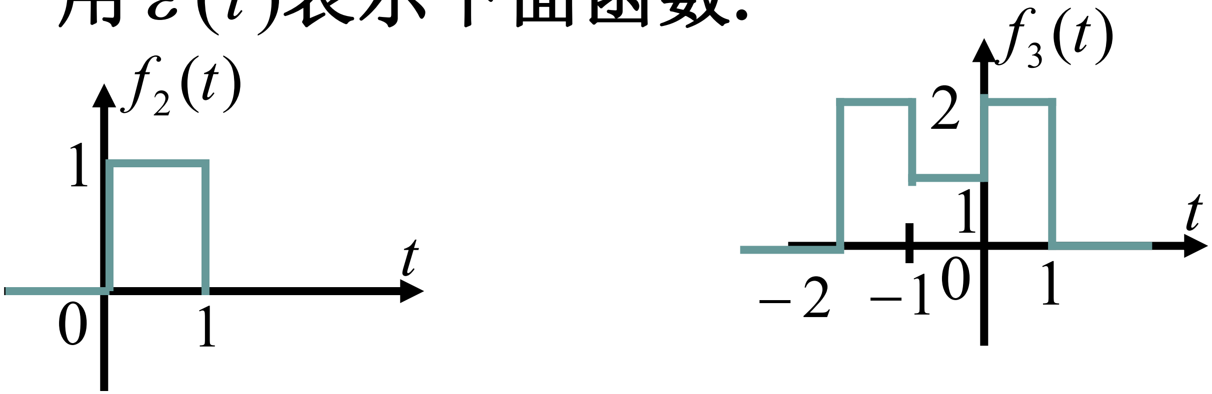
法二：



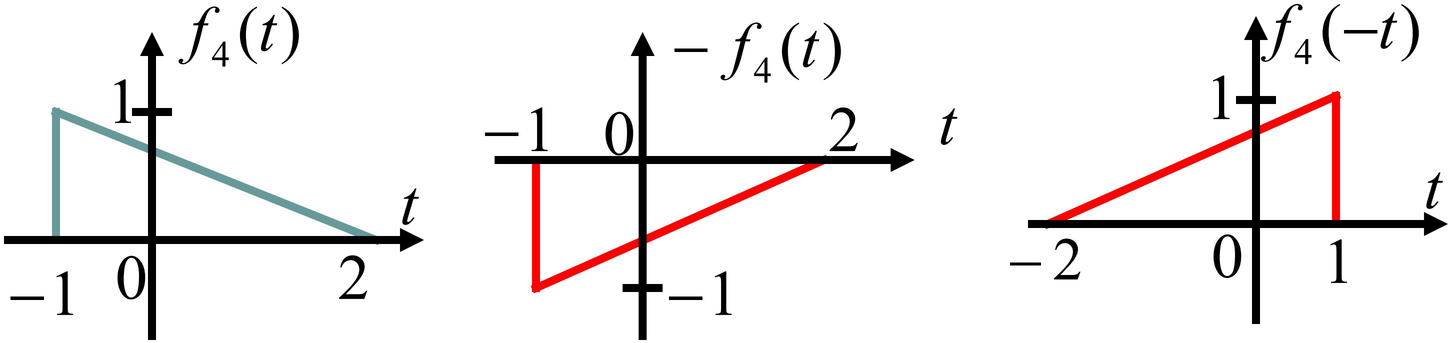
练习1



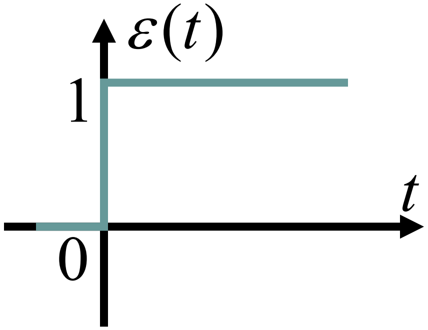
练习2 用$\varepsilon(t)$表示下面函数.



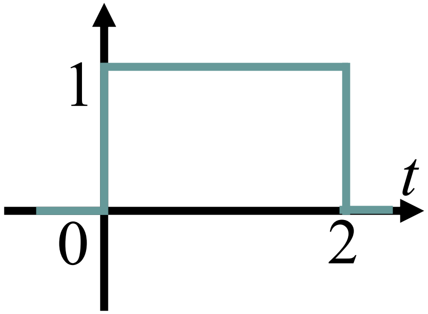
练习3



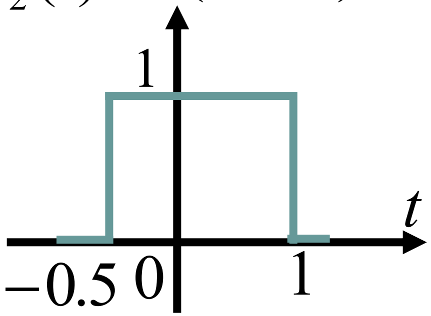
练习4



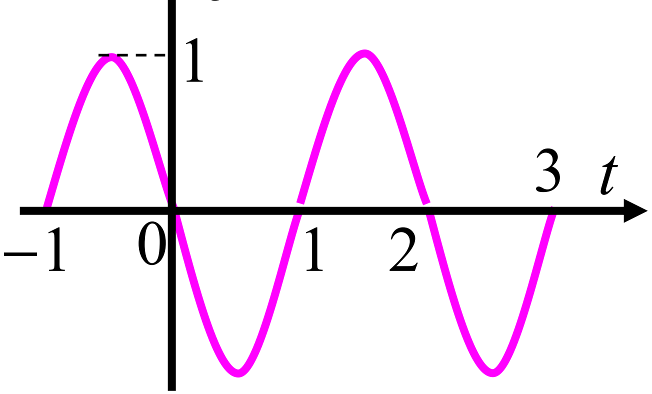
$f_1(t) = \varepsilon(t) - \varepsilon(t-2)$



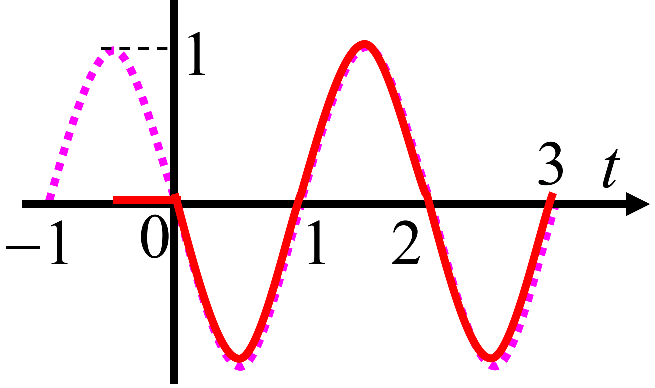
$f_2(t) = \varepsilon(t+0.5) - \varepsilon(t-1)$



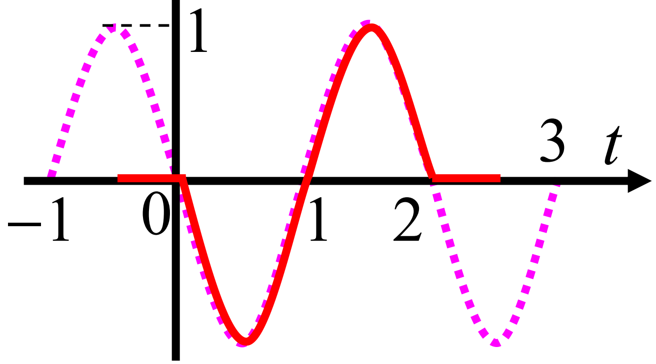
$f_3(t) = -\sin(\pi t)$



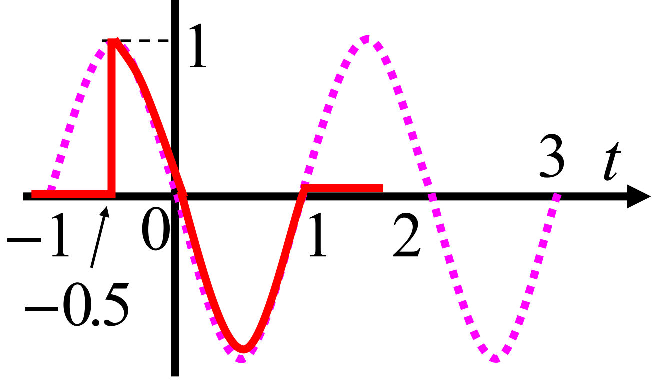
$f_4(t) = f_3(t)\varepsilon(t)$



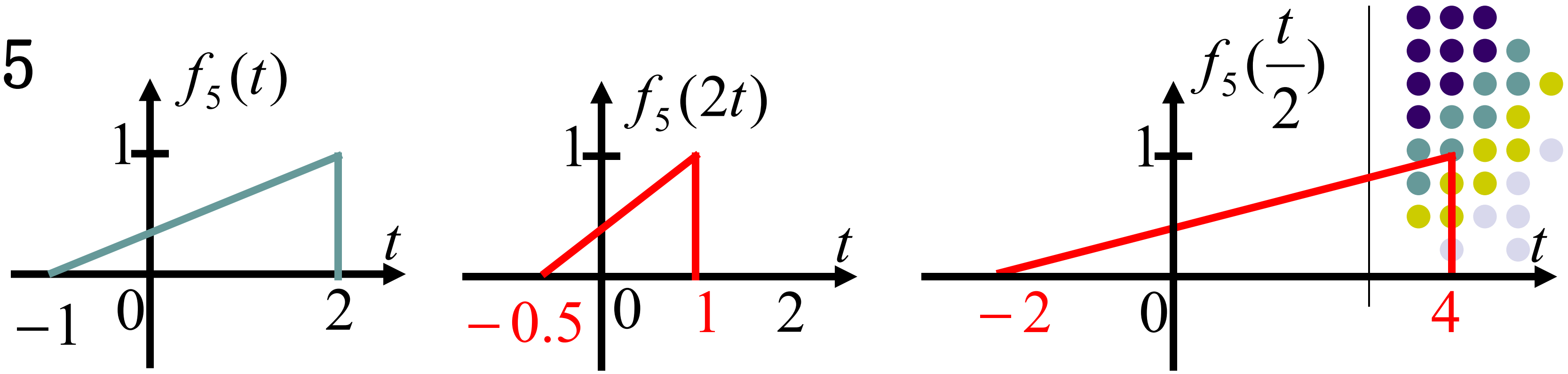
$f_5(t) = f_3(t) f_1(t)$



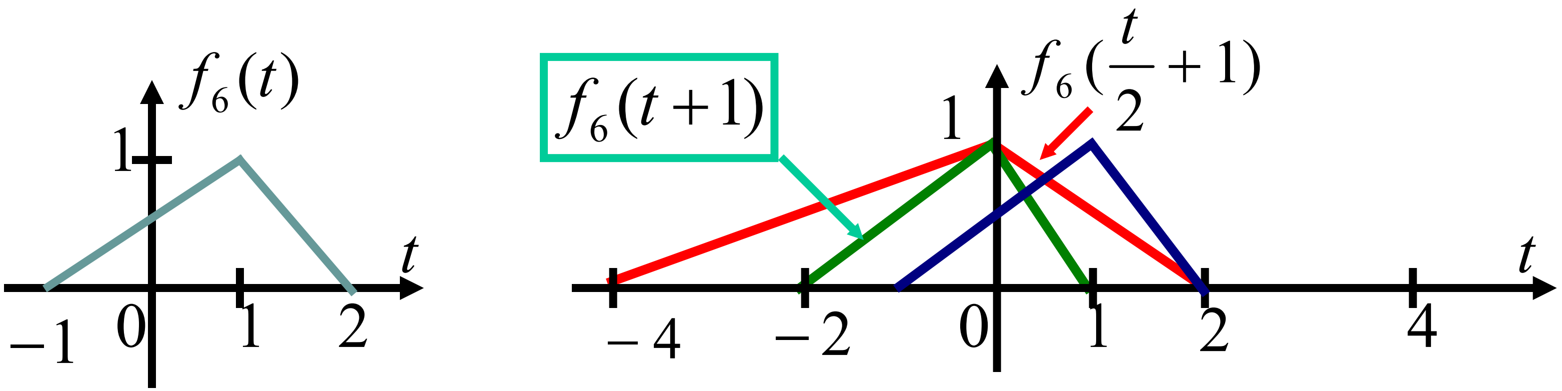
$f_6(t) = f_3(t) f_2(t)$



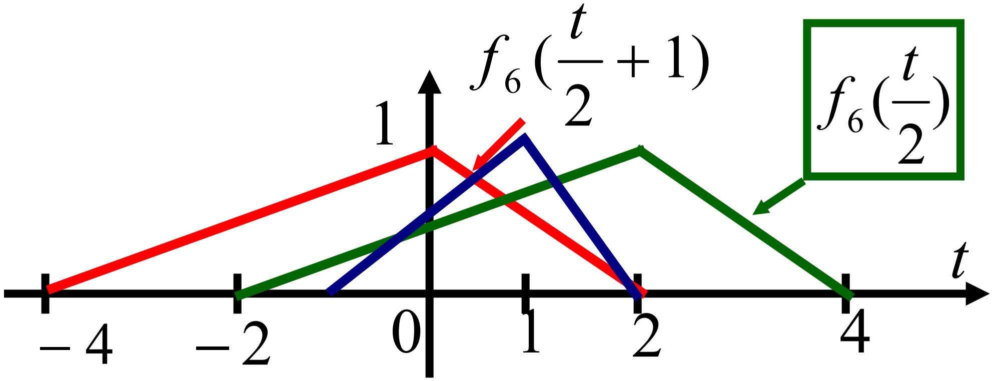
练习5



练习6



画 $f_6(\frac{t}{2}+1)$ 波形

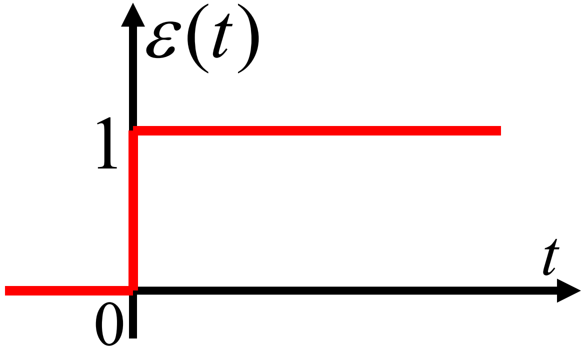


1.2 阶跃函数和冲激函数(§ 1.4)

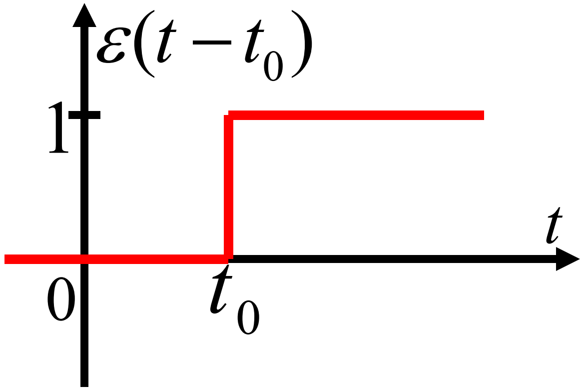
一. 阶跃函数 $\varepsilon(t)\left[u(t)\right]$ *Unit step signal*

1. 定义

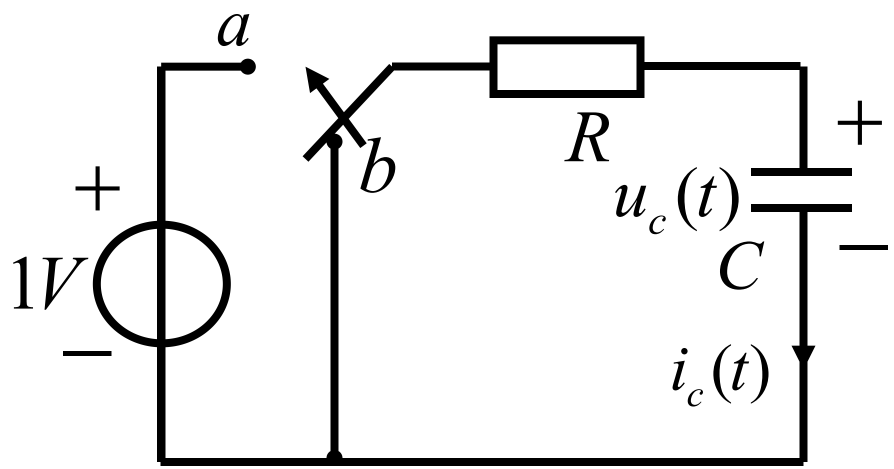
$$\varepsilon(t)=\begin{cases}1, t>0\\ \frac{1}{2}, t=0\\ 0, t<0\end{cases}$$

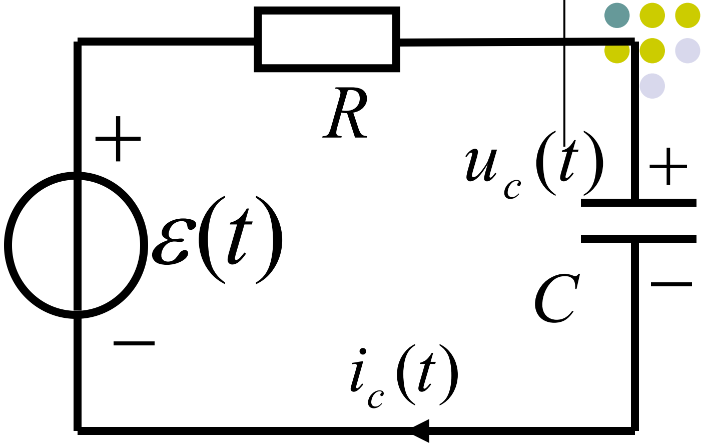


$$\varepsilon(t-t_0)=\begin{cases}1, t>t_0\\ \frac{1}{2}, t=t_0\\ 0, t<t_0\end{cases}$$



2. 实现(物理模型)





$$u_c(t) = \left[1 - e^{-\frac{t}{RC}}\right]\underline{\varepsilon(t)}$$

$$C = 1F, \quad R \longrightarrow 0$$

$$u_c(t) = \varepsilon(t)$$

$$i_c(t) = \frac{1}{R}e^{-\frac{t}{RC}}.\underline{\varepsilon(t)}$$

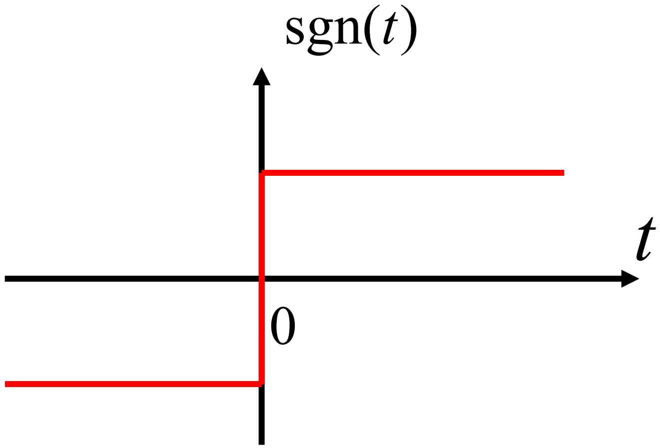
$$i_c(t) = C\frac{d}{dt}u_c(t) = \delta(t)$$

3. $\varepsilon(t)$ 应用举例

（1）方便地表示其它函数

例　符号函数 $\mathrm{sgn}(t)$

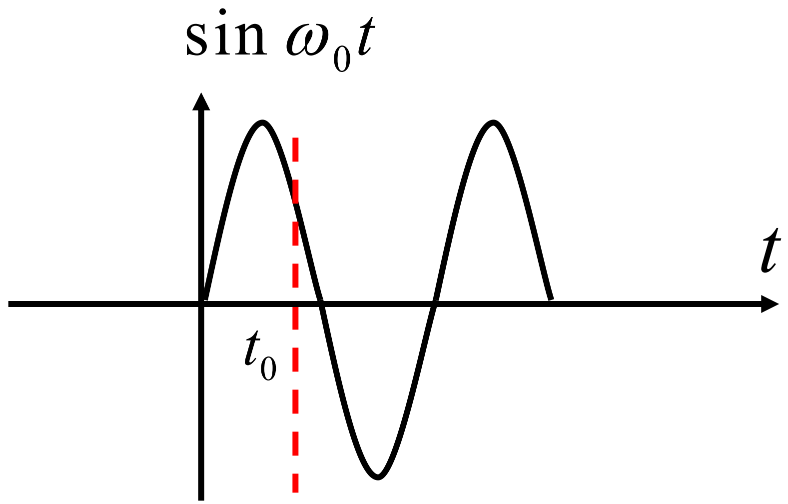
$$\mathrm{sgn}(t)=\begin{cases}-1, t<0\\ 0, t=0\\ 1, t>0\end{cases}$$

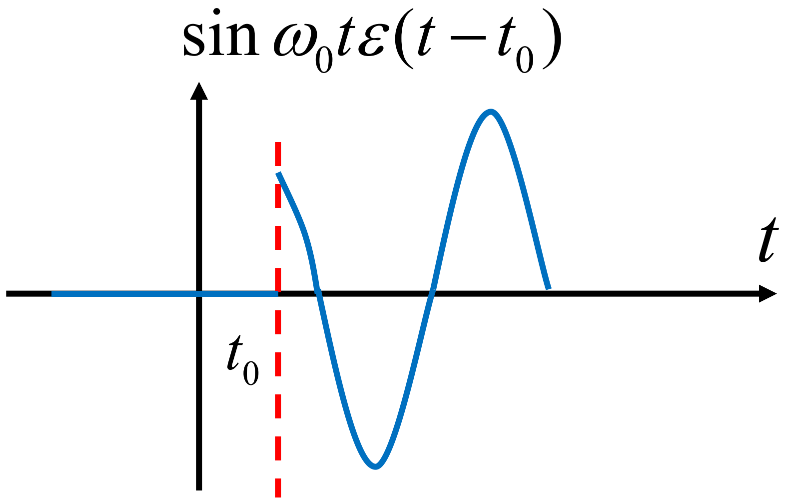


$$\mathrm{sgn}(t)=2\varepsilon(t)-1$$

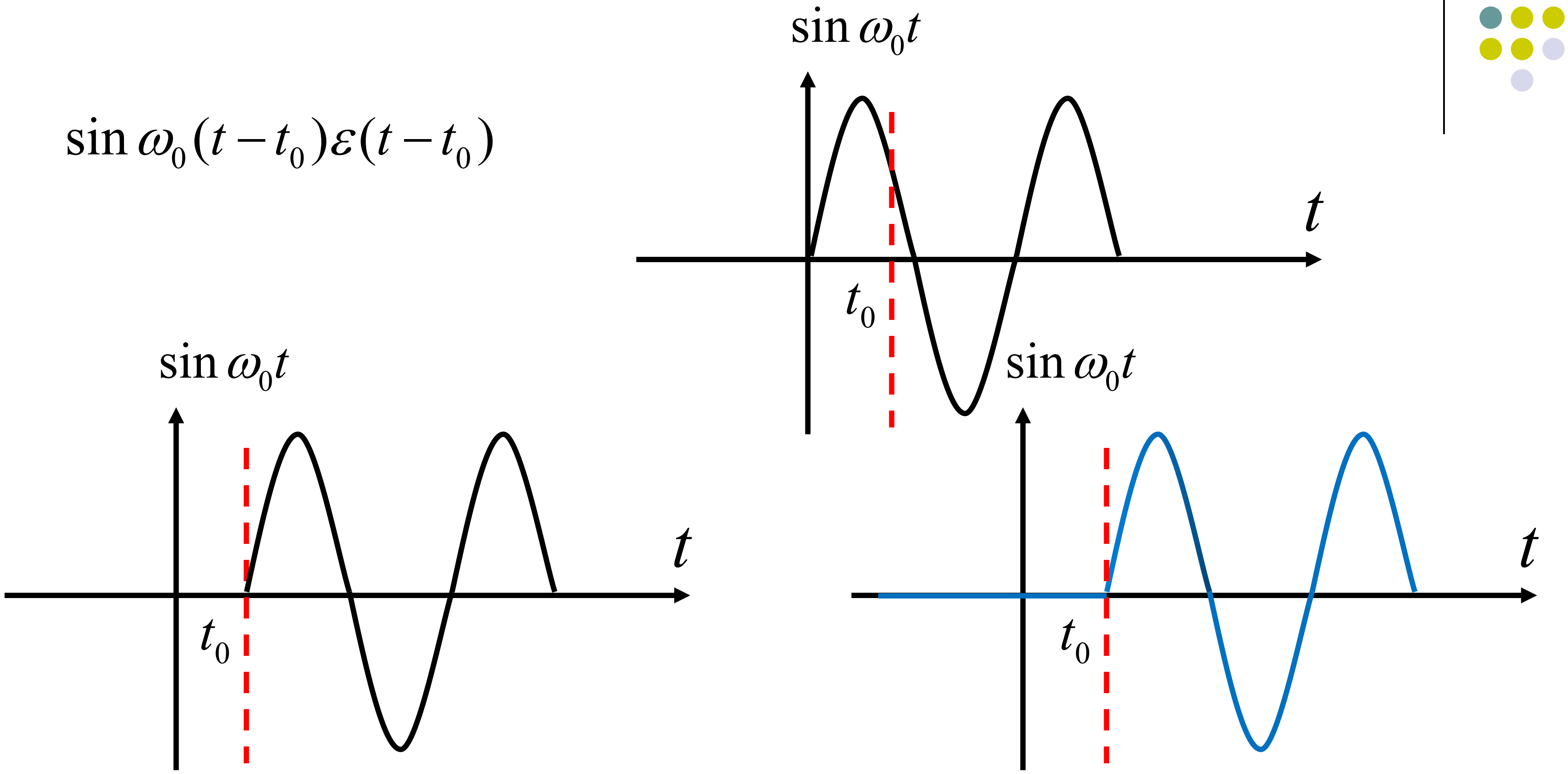
（2）用阶跃函数表示信号的作用区间

$$\sin\omega_0 t\varepsilon(t-t_0)$$



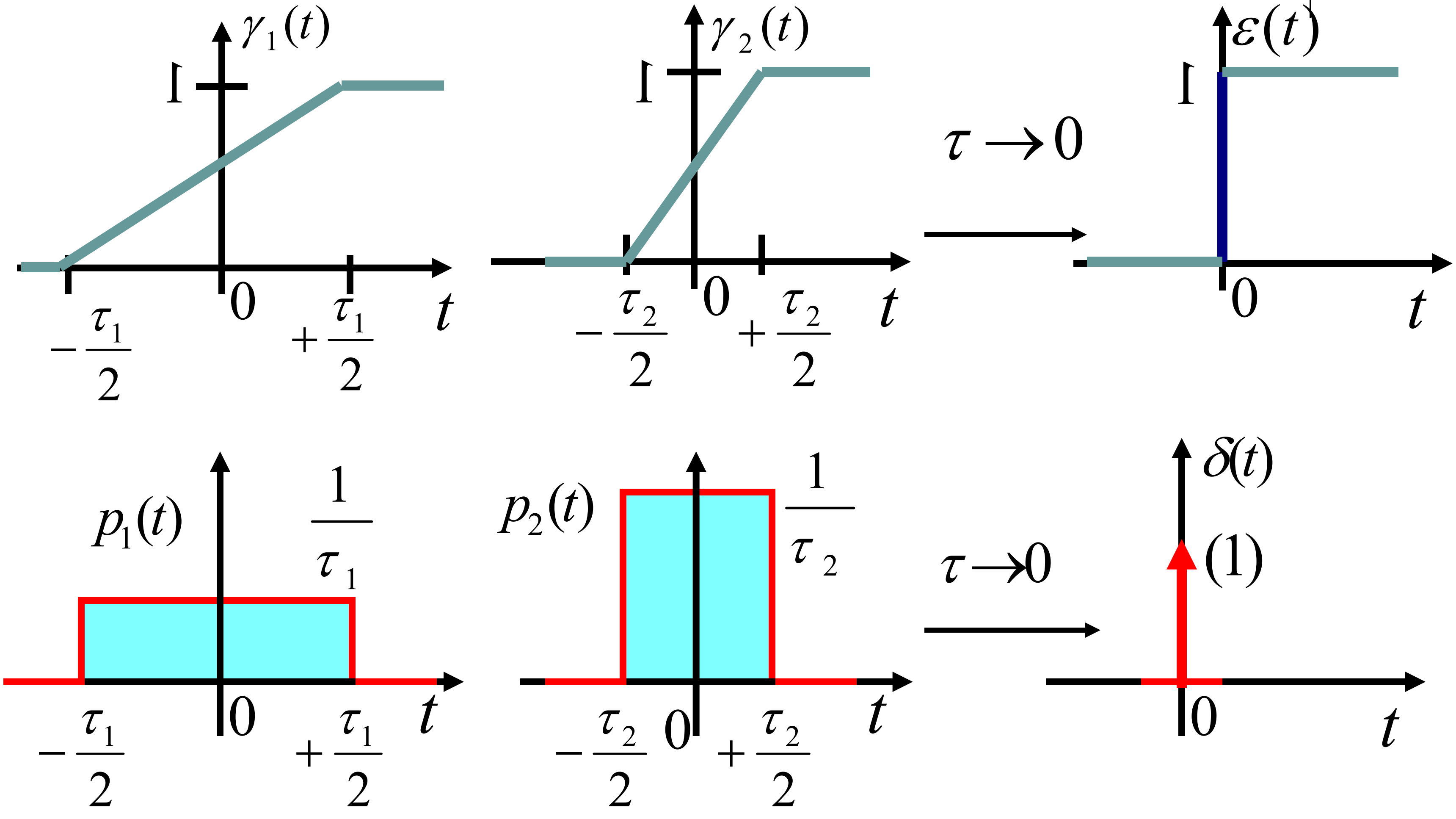


$$\sin \omega_0 (t - t_0)\varepsilon(t - t_0)$$



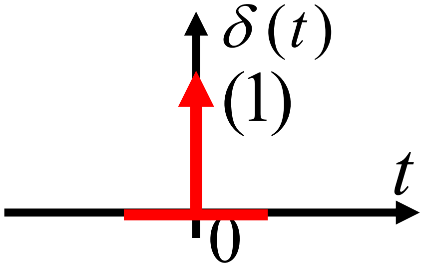
二. 冲激函数 $\delta(t)$ *Unit Impulse Signal*.

1. 定义1 (视为一般函数序列的极限)

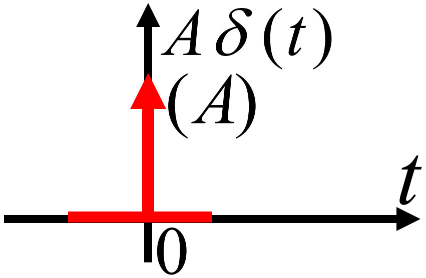


2. 定义2 狄拉克(Dirac)函数

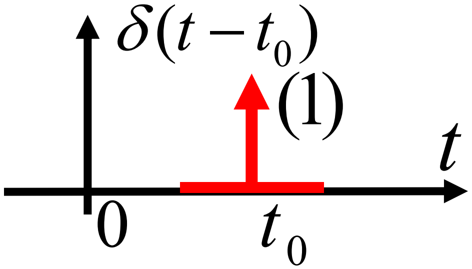
$$\delta(t):\begin{cases}\delta(t)=0,t\neq 0\\ \int_{-\infty}^{+\infty}\delta(t)dt=1\end{cases}$$



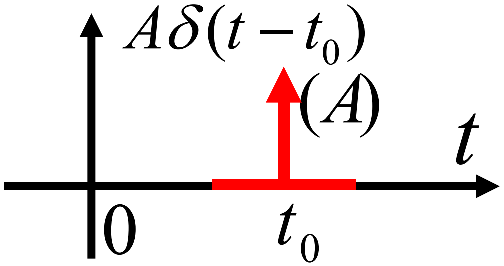
$$A\delta(t):\begin{cases}\delta(t)=0,t\neq 0\\ \int_{-\infty}^{+\infty}A\delta(t)dt=A\end{cases}$$



$$\delta(t-t_0):\begin{cases}\delta(t-t_0)=0,t\neq t_0\\ \int_{-\infty}^{+\infty}\delta(t-t_0)dt=1\end{cases}$$



$$A\delta(t-t_0):\begin{cases}A\delta(t-t_0)=0,t\neq t_0\\ \int_{-\infty}^{+\infty}A\delta(t-t_0)dt=A\end{cases}$$



三. $\delta(t)$ 函数的性质

1. $\delta(t)=\frac{d}{dt}\varepsilon(t)$ $\quad$ $\varepsilon(t)=\int_{-\infty}^{t}\delta(\tau)d\tau$

$\delta(t-t_0)=\frac{d}{dt}\varepsilon(t-t_0)$ $\quad$ $\varepsilon(t-t_0)=\int_{-\infty}^{t}\delta(\tau-t_0)d\tau$

2. $\delta(t)=\delta(-t)$

3. $x(t)\delta(t)=x(0)\delta(t)$

例1.2-1 计算(P19例1.4-1)

$$t\delta(t)=0$$

$$e^{-\alpha t}\delta(t)=\delta(t)$$

$$\delta(t)\cos t=\delta(t)$$

$$\delta(t)\sin 3t=0$$

4. $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

例 1.2-2 计算 $\cos t\,\delta(t-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}\delta(t-\frac{\pi}{4})$

$$\delta(t+\frac{\pi}{4})\sin t = -\frac{\sqrt{2}}{2}\delta(t+\frac{\pi}{4})$$

5. $\int_{-\infty}^{+\infty} x(t)\delta(t)dt = x(0)$ $\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$

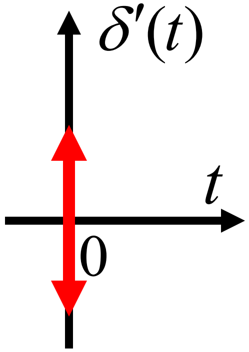
例 1.2-3 计算 $\int_{-\infty}^{+\infty} 2t\delta(t-1)dt = 2$

$$\int_{-\infty}^{+\infty} \delta(t+3)e^{-t}dt = e^3$$

练习 计算 $\int_{-\infty}^{+\infty} \delta(1-t)(t^3+4)dt = 5$

6. $\delta(t)$ 的导数

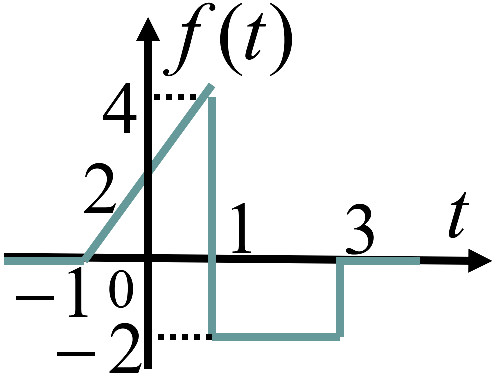
$$\delta'(t) = \frac{d}{dt}\delta(t) \qquad \int_{-\infty}^{+\infty}\delta'(t)dt = 0$$

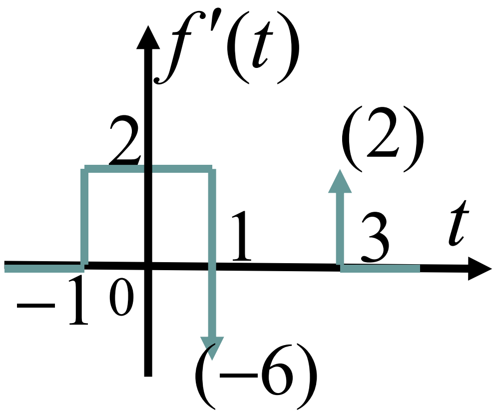


$$\int_{-\infty}^{+\infty}\delta'(t)x(t)dt = -x'(0)$$

例 1.2-4 计算 $\frac{d}{dt}\left[\cos(\omega_0 t).\varepsilon(t)\right] = \delta(t) - \omega_0\sin(\omega_0 t)\varepsilon(t)$

例1.2-5化出 $f'(t)$ 的波形图(P20例1.4-2)





$$f(t)=(2t+2)\left[\varepsilon(t+1)-\varepsilon(t-1)\right]-2\left[\varepsilon(t-1)-\varepsilon(t-3)\right]$$

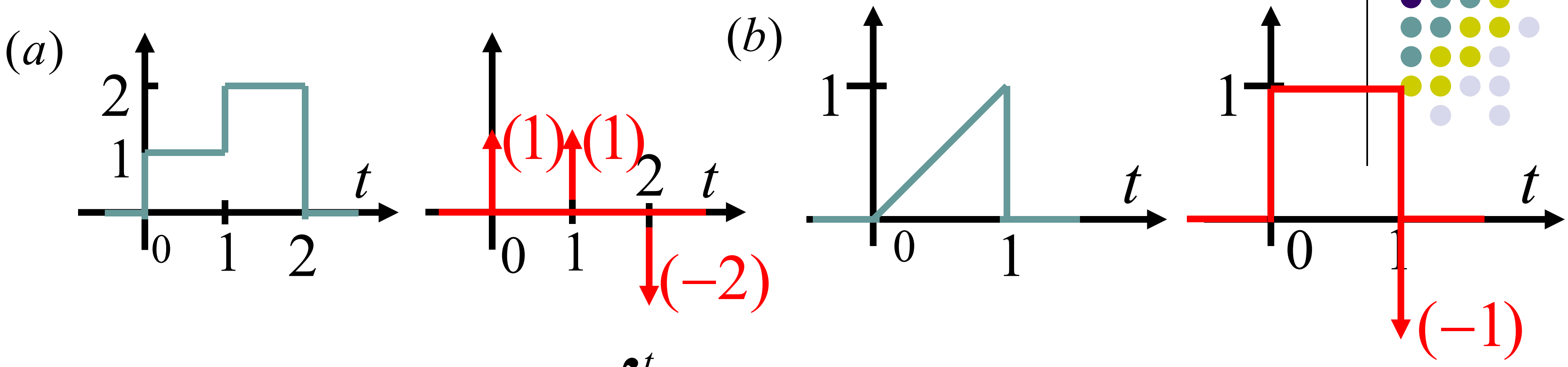
7.尺度变换 $\delta(at)=\dfrac{1}{|a|}\delta(t)$

8.卷积 (1). $f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}f_1(\tau)f_2(t-\tau)d\tau$

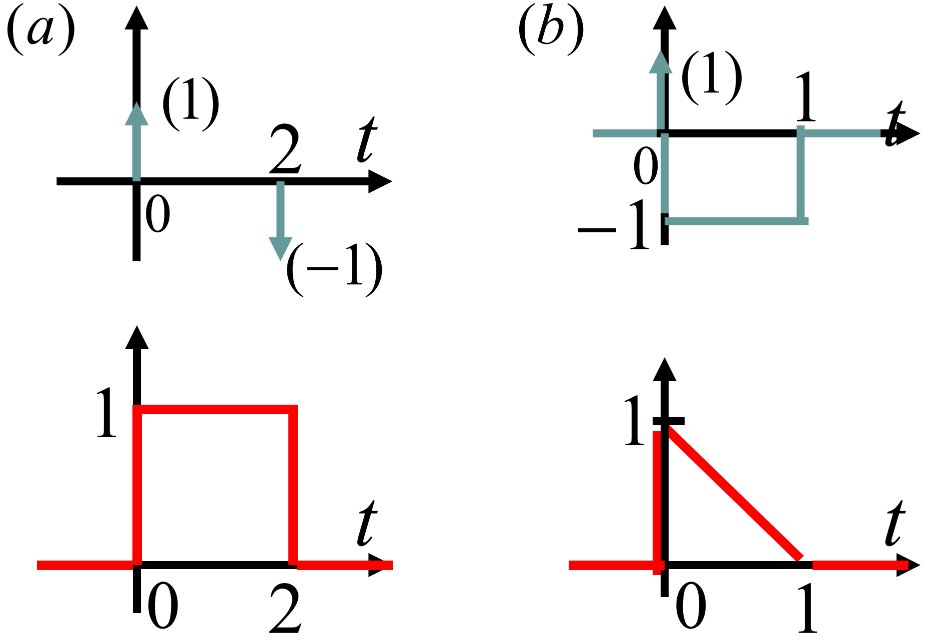
(2). $x(t)*\delta(t)=\delta(t)*x(t)=x(t)$

(3). $x(t)*\delta(t-t_0)=x(t-t_0)$

练习：求各函数的一阶导数



练习：求各函数的积分 $\int_{-\infty}^{t} f(x)dx$



1.3 系统的特性、分析和描述

一. 系统的分类（§1.6）

continuous time system

1. (时间)连续系统&离散系统&混合系统 *discrete ~ ~*

2. 因果系统&非因果系统 *causal system*

因果系统(物理可实现系统)：

若当 $t<0$ 时, $f(t)=0$, 则当 $t<0$ 时,一定有 $y(t)=0$.

3. 稳定系统&非稳定系统

稳定系统： 若 $|f(t)|<\infty$ 则 $|y_{zs}(t)|<\infty$

4. 非时变系统&时变系统 *time invariant system*

非时变系统： 若 $f(t)\rightarrow y(t)$ 则 $f(t-t_0)\rightarrow y(t-t_0)$

5. 线性系统 & 非线性系统

linear system

(1).同时满足齐次性和可加性的系统,即若:

$$f_1(t) \to y_1(t), f_2(t) \to y_2(t)$$

则 $a_1 f_1(t) + a_2 f_2(t) \to a_1 y_1(t) + a_2 y_2(t)$

(2).可分解性: $y(t) = y_{zi}(t) + y_{zs}(t)$

$y_{zi}(t)$, $y_{zs}(t)$ 均满足线性.

$$y_x(t) \leftrightarrow y_{zi}(t)$$

$$y_f(t) \leftrightarrow y_{zs}(t)$$

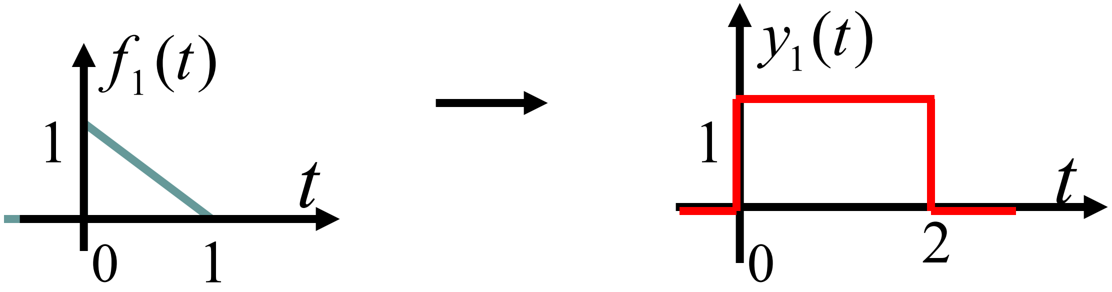
6. 线性非时变系统（LTI）性质推论

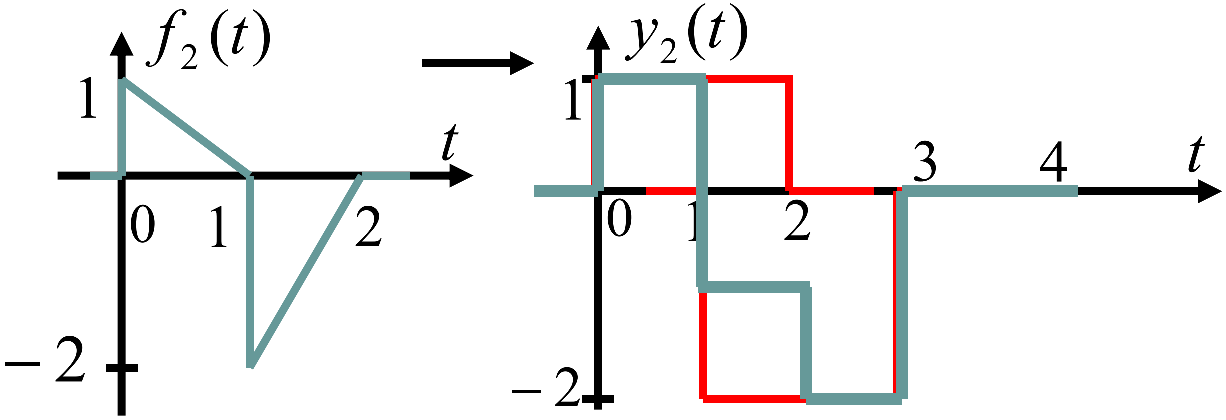
若 $f(t) \to y(t)$ *linear time invariant system*

则

$$\boxed{\begin{array}{l} af(t) \to ay(t) \\ f'(t) \to y'(t) \\ \int_{-\infty}^{t} f(x)dx \to \int_{-\infty}^{t} y(x)dx \end{array}}$$

例1.3-1





linear

time invariant system

例1.3-2 设LTI系统初始状态 $x(0)$**不为零,输入** $f(t)$**时, 全响应**

$$y(t)=y_{zi}(t)+y_{zs}(t)$$

若初始状态不变,输入为 $2f(t)$ **,全响应**

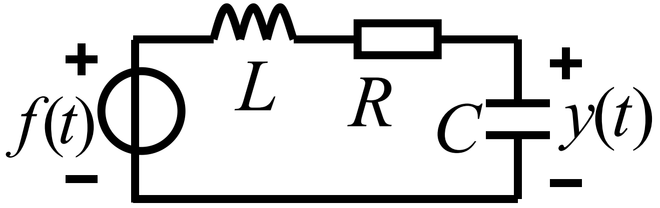
$$y(t)=y_{zi}(t)+2y_{zs}(t)$$

二. 系统的数学模型—方程（组）描述

（§1.5 系统的描述）

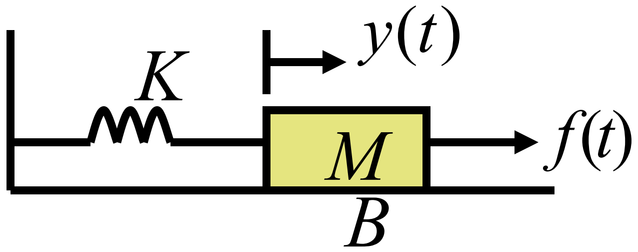
连续系统　线性常微分方程 ***Differential Equation***

例1.3-3



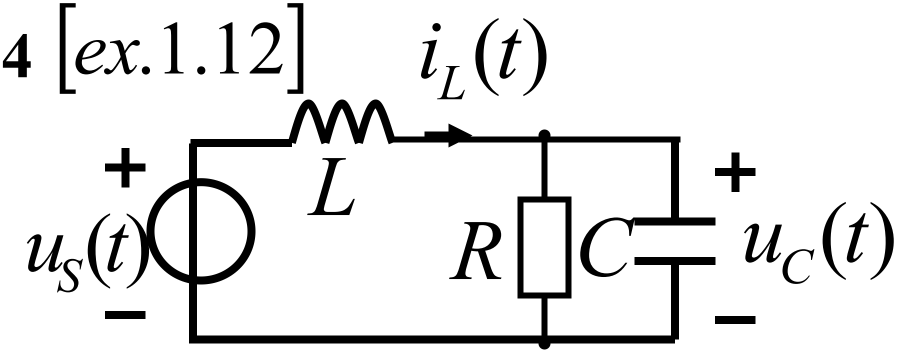
$$LCy''(t)+RCy'(t)+y(t)=f(t)$$

相似系统　具有相同的数学模型的系统.



$$My''(t)+By'(t)+Ky(t)=f(t)$$

例1.3-4 $\left[ex.1.12\right]$



输入为 $u_S(t)$, **分别以** $u_C(t)$, $i_L(t)$ **为输出(响应), 建立方程**

$$LCu''_C(t)+\frac{L}{R}u'_C(t)+u_C(t)=u_S(t)$$

$$u''_C(t)+\frac{1}{RC}u'_C(t)+\frac{1}{LC}u_C(t)=\frac{1}{LC}u_S(t)$$

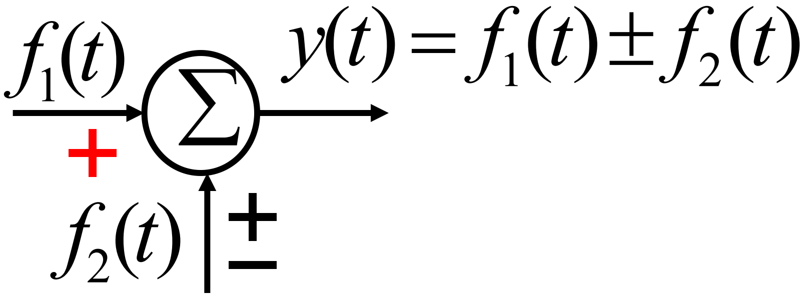
$$LCi''_L(t)+\frac{L}{R}i'_L(t)+i_L(t)=\frac{1}{R}u_S(t)+Cu'_S(t)$$

$$i''_L(t)+\frac{1}{RC}i'_L(t)+\frac{1}{LC}i_L(t)=\frac{1}{RLC}u_S(t)+\frac{1}{L}Cu'_S(t)$$

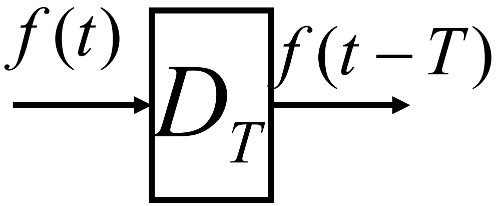
三. 系统的框图表示

1. 基本部件

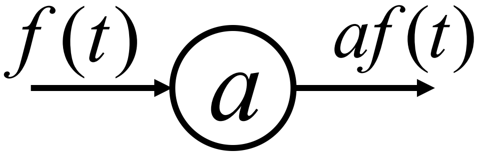
求和器

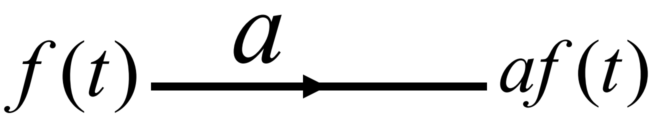


延迟器

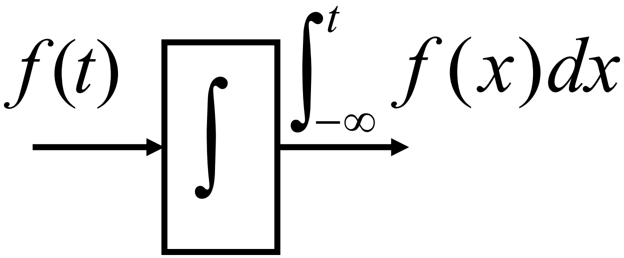


数乘器

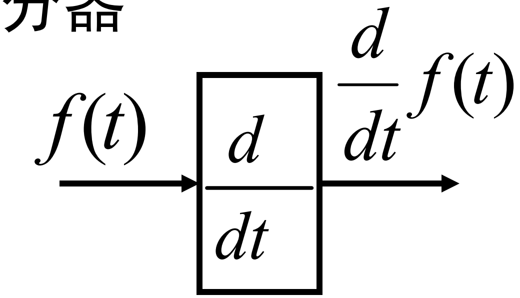




积分器

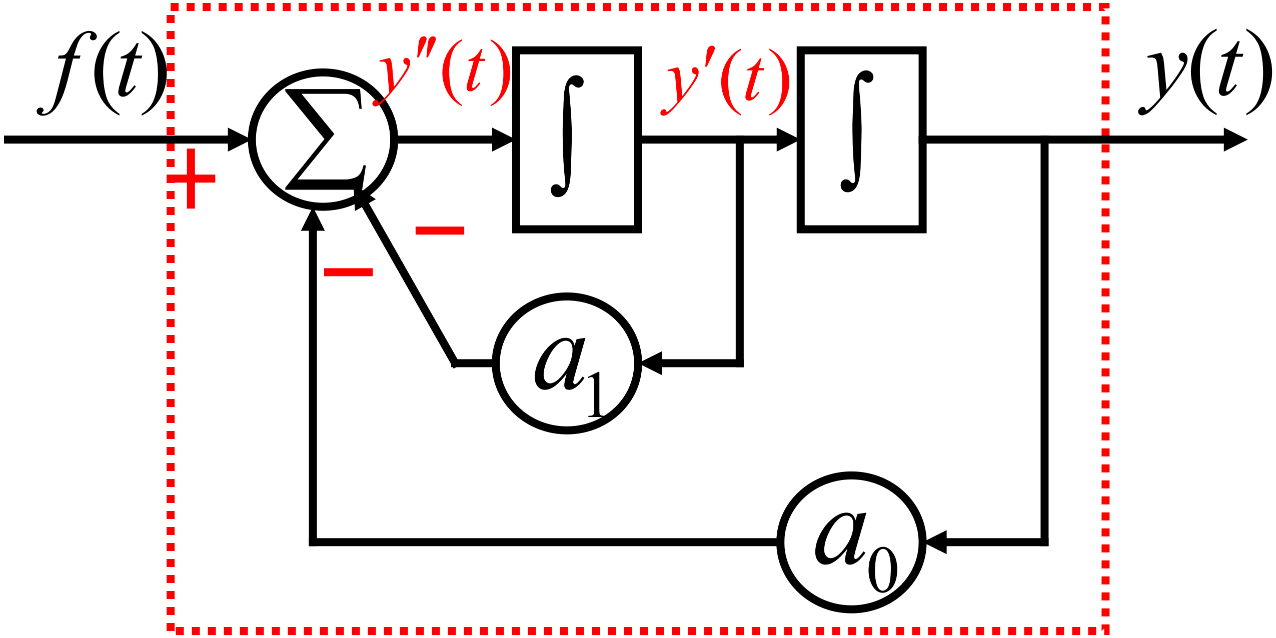


微分器



2. 由框图写微分方程

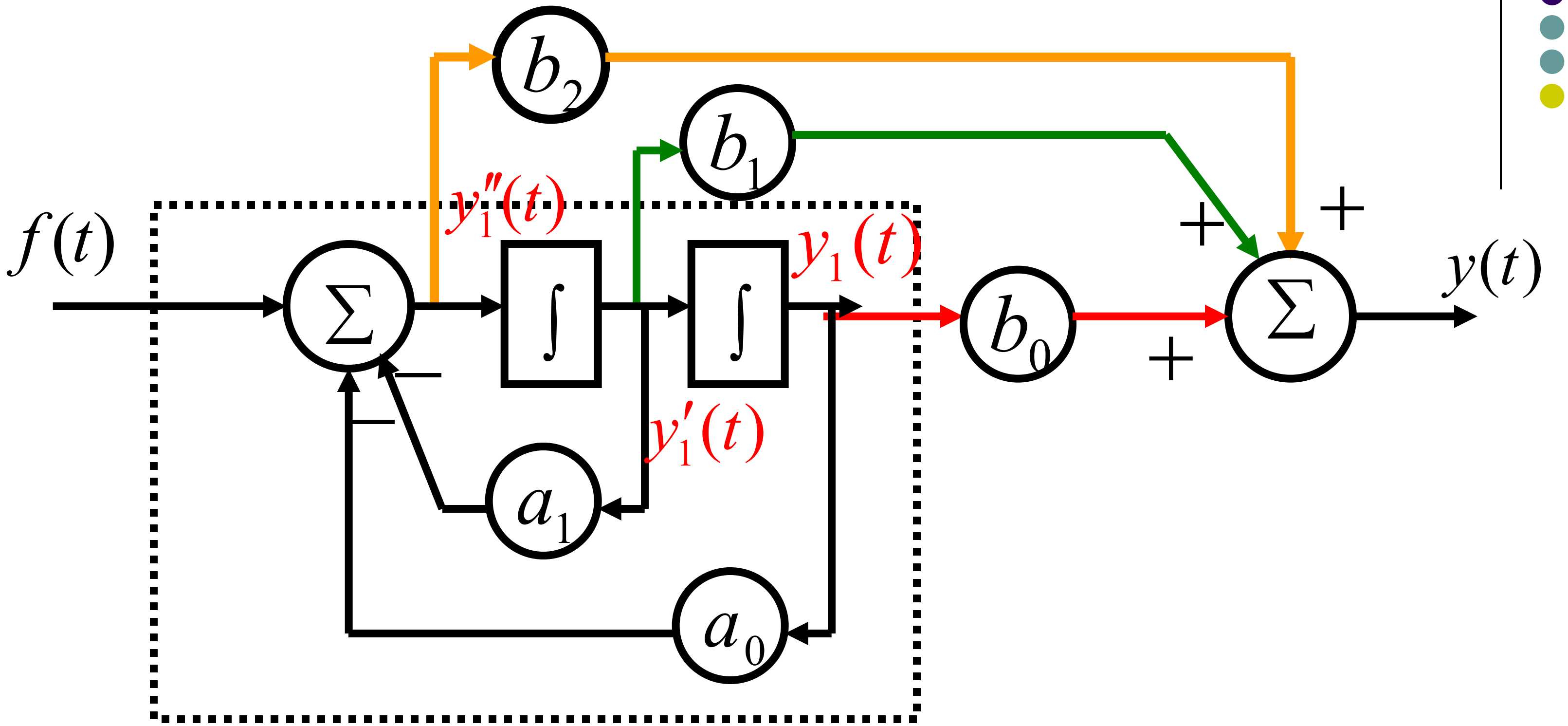
例1.3-5 写出描述系统的微分方程.（P25例1.5-1)



$$y''(t) = -a_1 y'(t) - a_0 y(t) + f(t) \quad (1-1)$$

$$y''(t) + a_1 y'(t) + a_0 y(t) = f(t)$$

例1.3-6 写出描述系统的微分方程.(P25例1.5-2)



$$y_1''(t) = -a_1 y_1'(t) - a_0 y_1(t) + f(t) \quad (1-1)$$

$$y_1''(t) + a_1 y_1'(t) + a_0 y_1(t) = f(t) \quad (1)$$

$$y''(t) + a_1 y'(t) + a_0 y(t) = b_2 f''(t) + b_1 f'(t) + b_0 f(t)$$

第二章 连续时间系统的时域分析

要点：

1. 微分方程的解
2. 单位冲激响应$h(t)$，
   单位阶跃响应$g(t)$
3. 卷积运算（定义，性质）
4. 连续时间系统的时域分析

2.1 卷积积分（§2.3 §2.4）

一. 卷积(积分)定义 *Convolution*

定义：　对于函数 $f_1(t), f_2(t), \quad -\infty < t < \infty$

定义卷积：　$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau).f_2(t-\tau)d\tau$

例2.1-1 已知： $f_1(t) = \varepsilon(t), f_2(t) = e^{-t}\varepsilon(t)$

$$f_3(t) = e^{-2t}\varepsilon(t)$$

计算：

$$y_1(t) = f_1(t) * f_2(t) = (1-e^{-t})\underline{\varepsilon(t)}$$

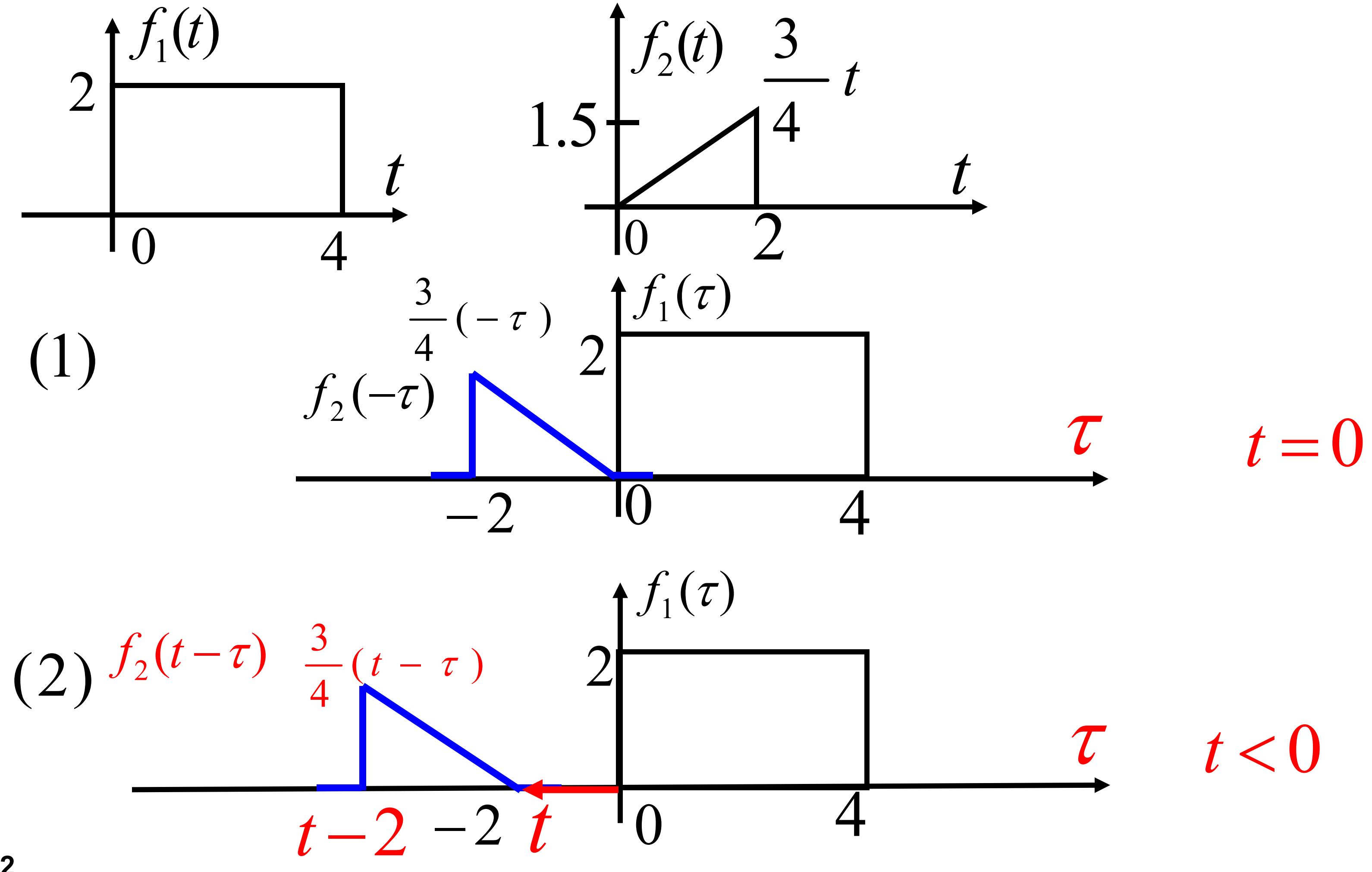
$$y_2(t) = f_2(t) * f_1(t) = (1-e^{-t})\underline{\varepsilon(t)}$$

$$y_3(t) = f_2(t) * f_3(t) = (e^{-t}-e^{-2t})\underline{\varepsilon(t)}$$

二. 卷积的图解

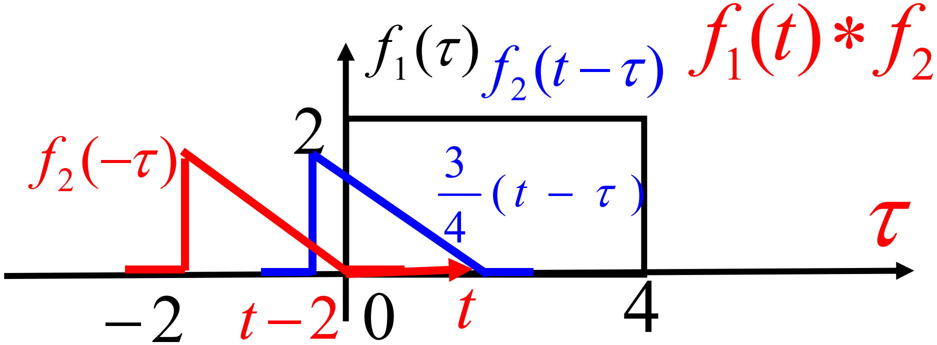
$$f_1(t)*f_2(t)=\int_{-\infty}^{\infty}f_1(\tau).f_2(t-\tau)d\tau$$

例2.1-2 (p61图2.3-3,图2.3-4) 计算 $y_1(t)=f_1(t)*f_2(t)$



$$f_1(t)*f_2(t)=\int_{-\infty}^{\infty} f_1(\tau).f_2(t-\tau)d\tau$$

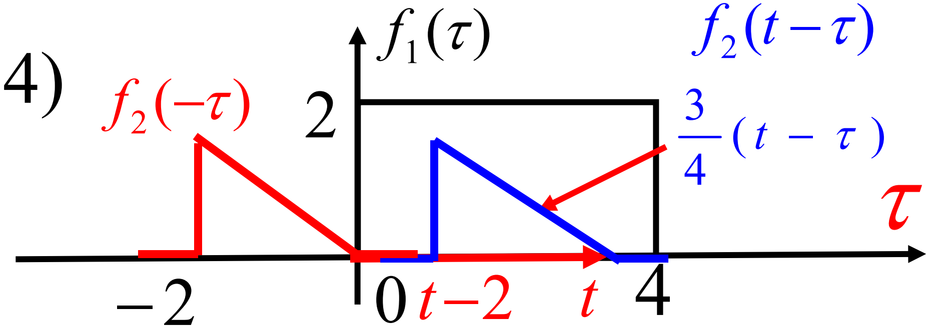
(3)



$$y(t)=\int_0^t \frac{3}{2}(t-\tau)d\tau=\frac{3}{4}t^2$$

$$2>t\geq 0$$

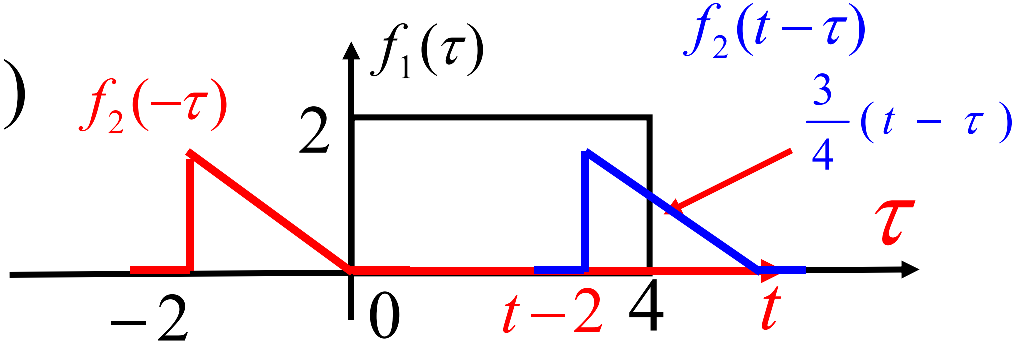
(4)



$$y(t)=\int_{t-2}^t \frac{3}{2}(t-\tau)d\tau=3$$

$$4>t\geq 2$$

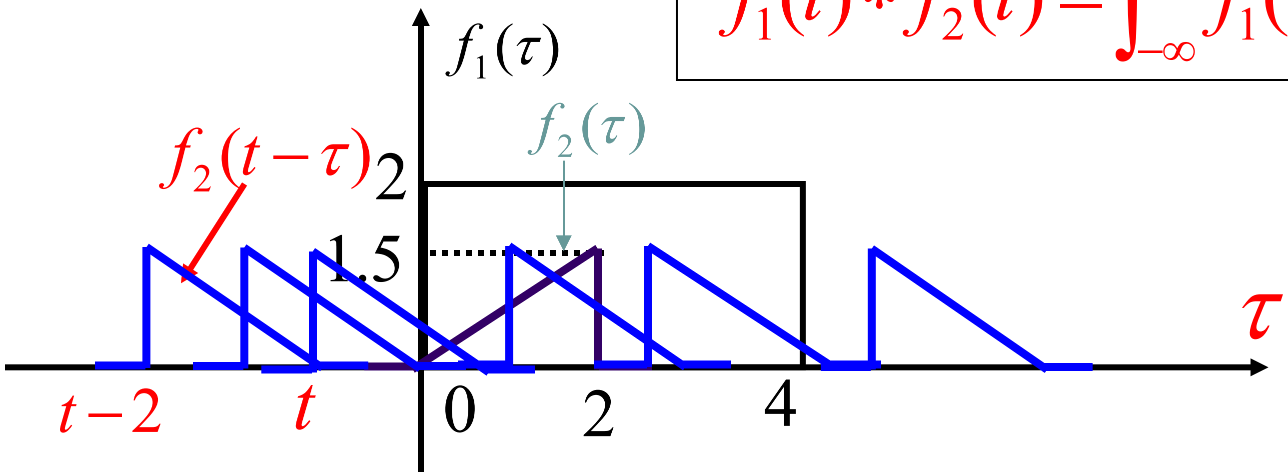
(5)



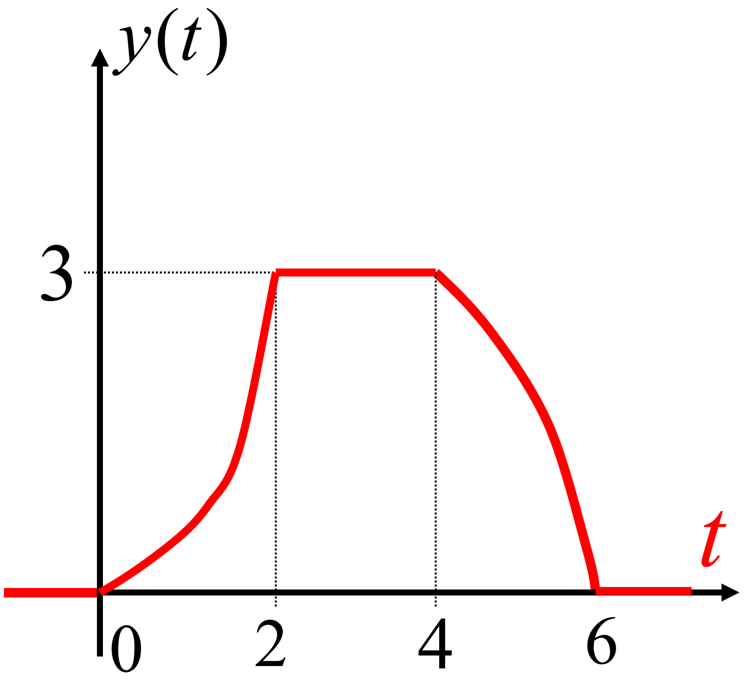
$$6>t\geq 4$$

$$y(t)=\int_{t-2}^4 \frac{3}{2}(t-\tau)d\tau=-\frac{3}{4}t^2+6t-9$$

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau).f_2(t-\tau)d\tau$$



$$y(t) = \begin{cases} 0 & , \quad t < 0 \\ \frac{3}{4}t^2 & , \quad 2 > t \geq 0 \\ 3 & , \quad 4 > t \geq 2 \\ -\frac{3}{4}t^2 + 6t - 9 & , \quad 6 > t \geq 4 \\ 0 & , \quad t \geq 6 \end{cases}$$



三. 卷积的运算性质

1.交换率: $f_1(t)*f_2(t)=f_2(t)*f_1(t)$

2.分配率: $f_1(t)*\left[f_2(t)+f_3(t)\right]=f_1(t)*f_2(t)+f_1(t)*f_3(t)$

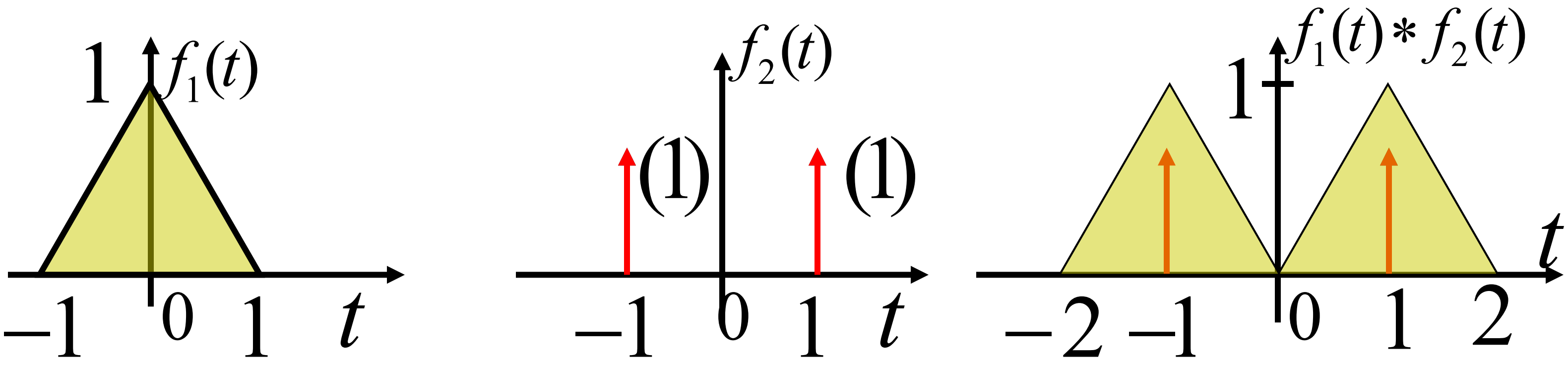
3.结合率: $f_1(t)*\left[f_2(t)*f_3(t)\right]=\left[f_1(t)*f_2(t)\right]*f_3(t)$

4. 冲激函数卷积:

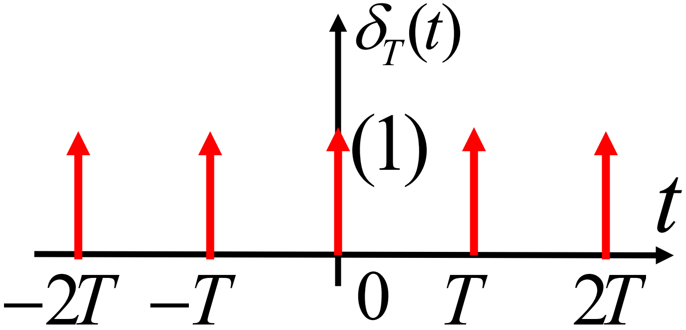
$$f(t)*\delta(t)=\delta(t)*f(t)=f(t)$$

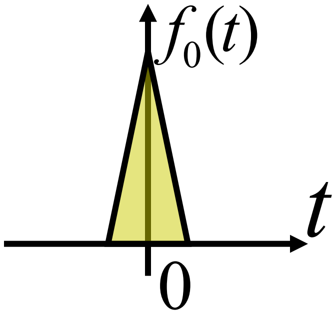
$$f(t)*\delta(t-t_0)=f(t-t_0)$$

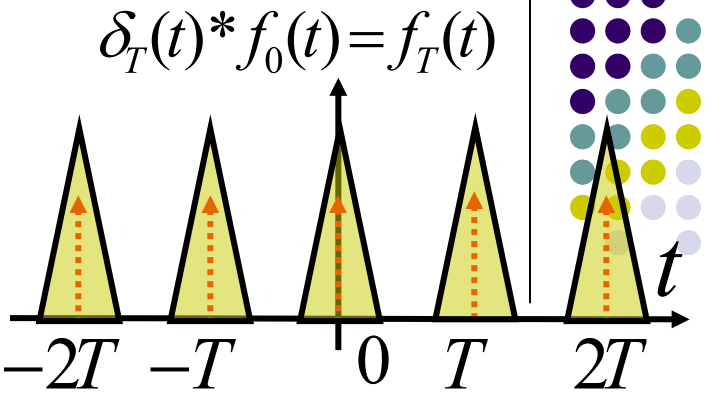
例2.1-3



例 2.1-4(P71例2.4-3)



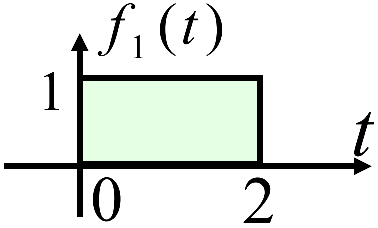


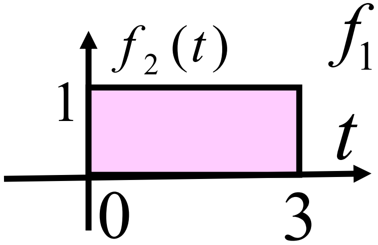


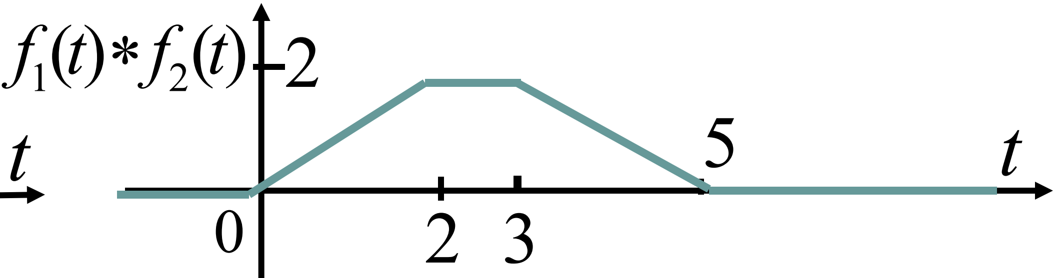
推论： 若 $f_1(t)*f_2(t)=y(t)$

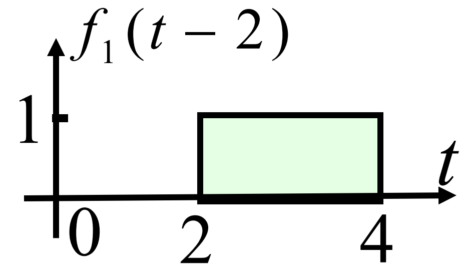
则 $f_1(t-t_1)*f_2(t-t_2)=y(t-t_1-t_2)$

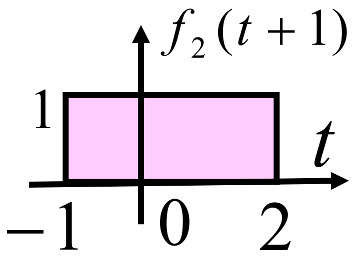
例2.1-5

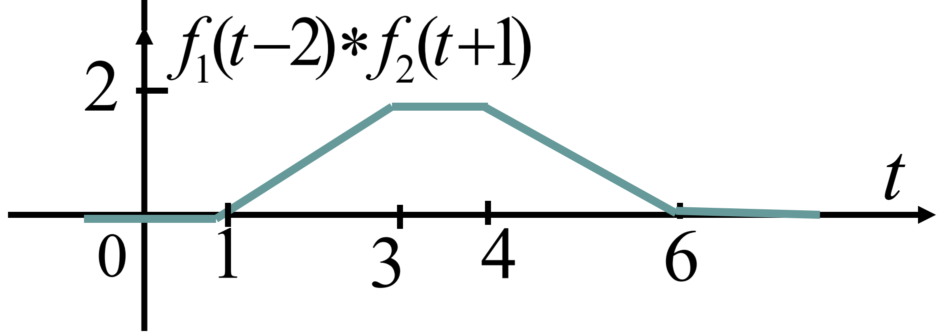










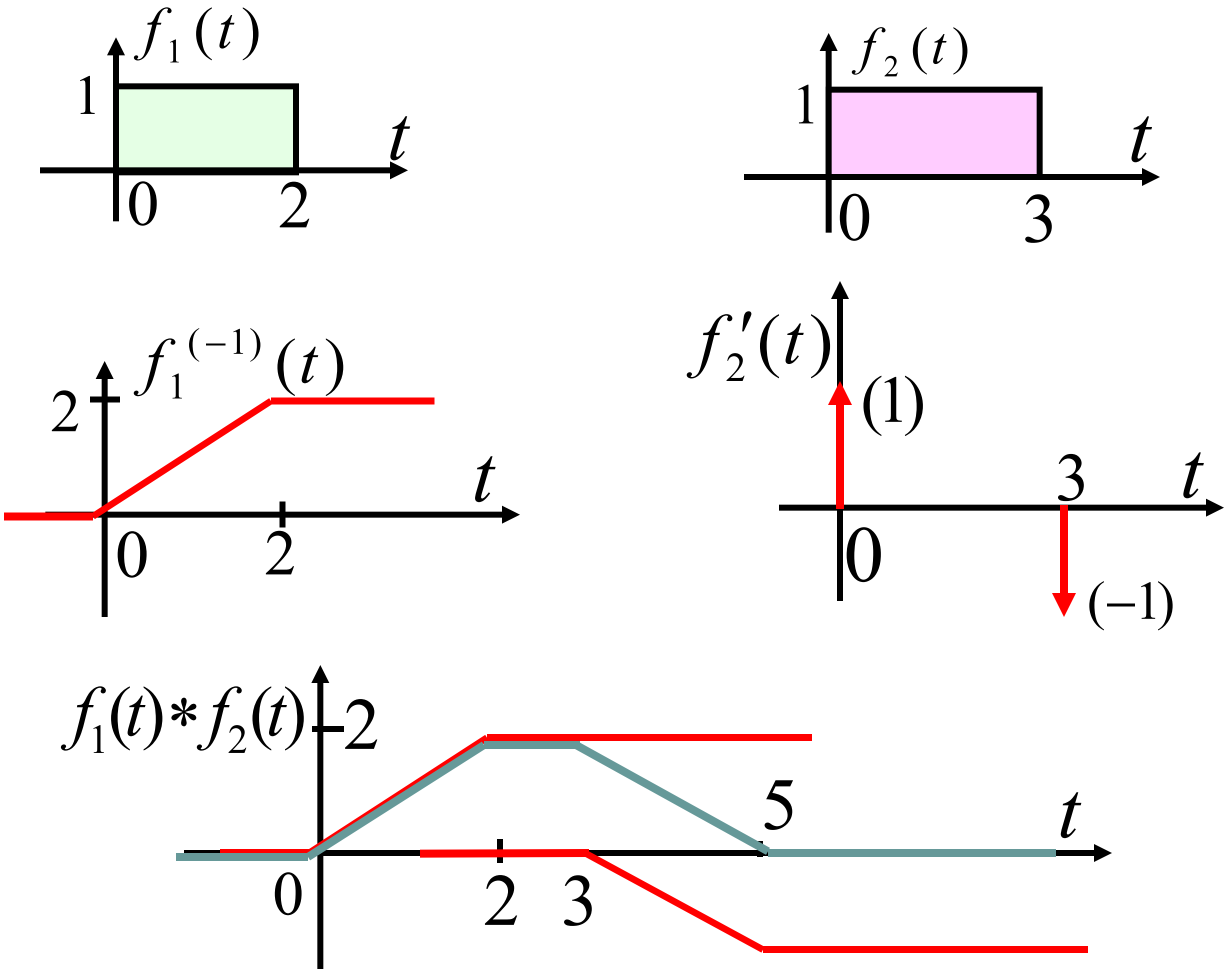


5.微分积分性质: 若 $y(t)=f_1(t)*f_2(t)$

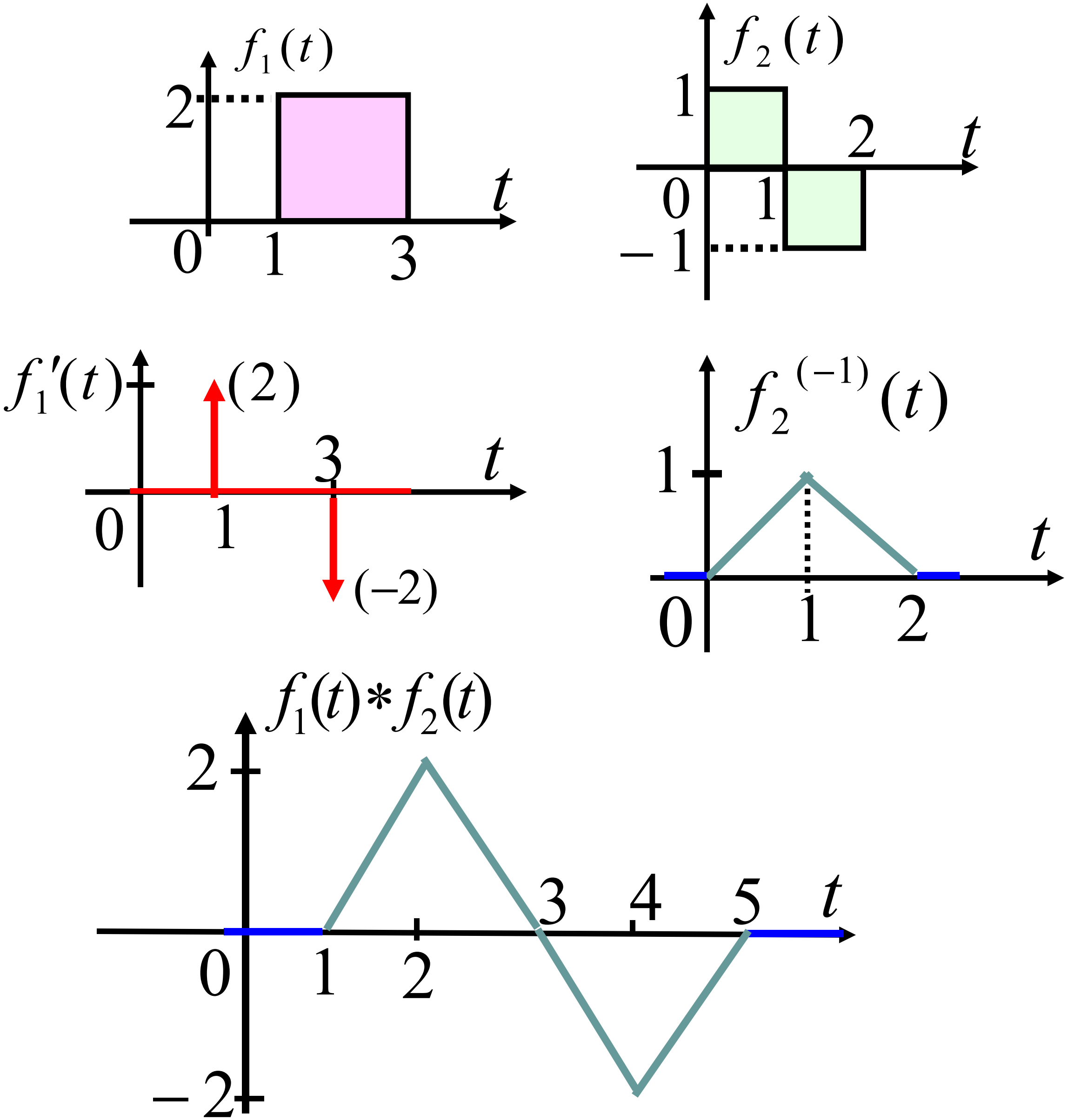
注

则 $y(t)=f_1^{(-1)}(t)*f_2'(t)=f_1'(t)*f_2^{(-1)}(t)$

例2.1-6



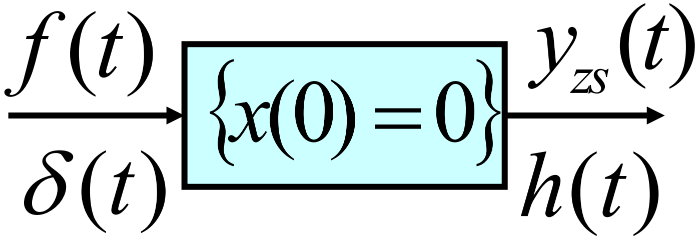
例2.1-7(P73例2.4-4)

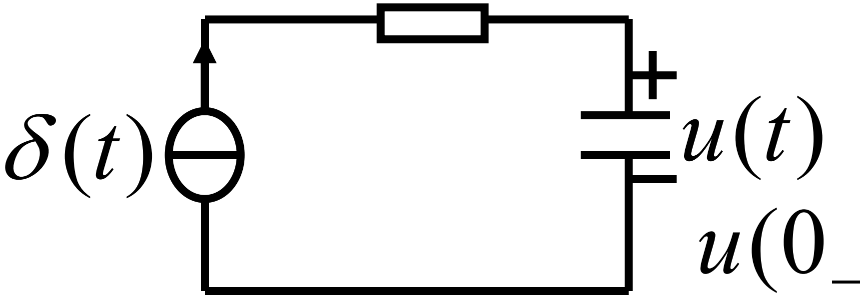


2.2　冲激响应和阶跃响应（§2.2）

一.（单位）冲激响应 $h(t)$

1.定义：　输入信号为 $\delta(t)$ 时的零状态响应。





$u(0\_)=0$

例　$$y''(t)+5y'(t)+6y(t)=f(t) \quad (1)$$

$$f(t)\rightarrow\delta(t)\,,y(t)\rightarrow h(t)$$

$$h''(t)+5h'(t)+6h(t)=\delta(t) \quad (2)$$

$$h''(t)+5h'(t)+6h(t)=0,t>0 \quad (3)$$

2. $h(t)$ 具有齐次解的形式

$$\Rightarrow h(t)=\left(C_1e^{-2t}+C_2e^{-3t}\right)\underline{\varepsilon(t)} \quad (4)$$

3. $h(t)$的求取 “匹配系数法”

例2.2-1

$$y''(t)+5y'(t)+6y(t)=f(t) \quad (1)$$

$$h''(t)+5h'(t)+6h(t)=\delta(t) \quad (2)$$

$$\Rightarrow h(t)=\left(C_1e^{-2t}+C_2e^{-3t}\right).\varepsilon(t) \quad (4)$$

$$h'(t)=(C_1+C_2)\delta(t)+(-2C_1e^{-2t}-3C_2e^{-3t}).\varepsilon(t)\ (5)$$

$$h''(t)=(C_1+C_2)\delta'(t)+(4C_1e^{-2t}+9C_2e^{-3t})\varepsilon(t)+(-2C_1-3C_2)\delta(t) \quad (6)$$

$$\begin{cases} (C_1+C_2)=0 \\ 5(C_1+C_2)+(-2C_1-3C_2)=1 \end{cases} \Rightarrow \begin{cases} C_1=1 \\ C_2=-1 \end{cases} \Rightarrow$$

$$h(t)=(e^{-2t}-e^{-3t})\underline{.\varepsilon(t)} \quad (7)$$

例 2.2-2 求 $h(t)$ $\quad y''(t)+5y'(t)+6y(t)=2f'(t)+3f(t) \quad (8)$

(a). "匹配系数法" $h''(t)+5h'(t)+6h(t)=2\delta'(t)+3\delta(t) \quad (9)$

令： $h(t)=(c_1e^{-2t}+c_2e^{-3t}).\varepsilon(t) \quad (10)$

$$h'(t)=(c_1+c_2)\delta(t)+(-2c_1e^{-2t}-3c_2e^{-3t}).\varepsilon(t) \quad (11)$$

$$h''(t)=(c_1+c_2)\delta'(t)$$

$$+(-2c_1-3c_2).\delta(t)+(4c_1e^{-2t}+9c_2e^{-3t})\varepsilon(t) \quad (12)$$

$$\begin{cases} c_1+c_2=2 \\ 5(c_1+c_2)+(-2c_1-3c_2)=3 \end{cases} \Rightarrow \begin{cases} c_1=-1 \\ c_2=3 \end{cases}$$

$$h(t)=(3e^{-3t}-e^{-2t})\varepsilon(t) \quad (13)$$

(b) 由线性时不变性质:

$$\underline{y''(t)+5y'(t)+6y(t)=f(t)} \quad (1)$$

$$\Rightarrow h_1(t)=(e^{-2t}-e^{-3t}).\varepsilon(t) \quad (7)$$

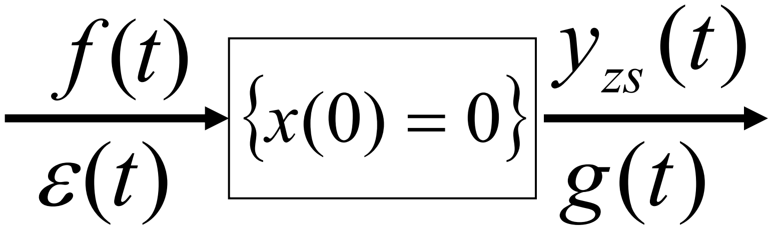
$$\underline{y''(t)+5y'(t)+6y(t)=2f'(t)+3f(t)} \quad (8)$$

$$\Rightarrow h(t)=2h_1'(t)+3h_1(t)$$

$$=\left(3e^{-3t}-e^{-2t}\right)\varepsilon(t) \quad (13)$$

二.（单位）阶跃响应

1. 定义



$f(t)$ / $\varepsilon(t)$ $\rightarrow$ $\{x(0)=0\}$ $\rightarrow$ $y_{zs}(t)$ / $g(t)$

2. $g(t)$的求取

依据: .线性时不变系统的性质

$$\varepsilon(t)=\int_{-\infty}^{t}\delta(x)dx \Rightarrow g(t)=\int_{-\infty}^{t}h(x)dx$$

例 求: $g(t)$ $$y''(t)+5y'(t)+6y(t)=f(t) \quad (1)$$

$$\Rightarrow h_1(t)=(e^{-2t}-e^{-3t}).\varepsilon(t) \quad (7)$$

$$g(t)=\int_{-\infty}^{t}(e^{-2x}-e^{-3x})\varepsilon(x)dx$$

$$g(t)=\left(\frac{1}{3}e^{-3t}-\frac{1}{2}e^{-2t}+\frac{1}{6}\right)\underline{\varepsilon(t)} \quad (14)$$

例　求(8)式所示系统的阶跃响应.

$$y''(t)+5y'(t)+6y(t)=2f'(t)+3f(t) \quad (8)$$

解:

$$y_1''(t)+5y_1'(t)+6y_1(t)=f(t) \quad (1)$$

$$\Rightarrow h_1(t)=(e^{-2t}-e^{-3t}).\varepsilon(t) \quad (7)$$

$$\Rightarrow h(t)=2h_1'(t)+3h_1(t)$$

$$=\left(3e^{-3t}-e^{-2t}\right)\varepsilon(t) \quad (13)$$

$$g(t)=\int_{-\infty}^{t}h(x)dx$$

$$=(\frac{1}{2}+\frac{1}{2}e^{-2t}-e^{-3t})\underline{.\varepsilon(t)}$$

2.3 LTI连续系统的响应（§2.1）

一. 微分方程的编写

二. 零输入响应

三. 零状态响应

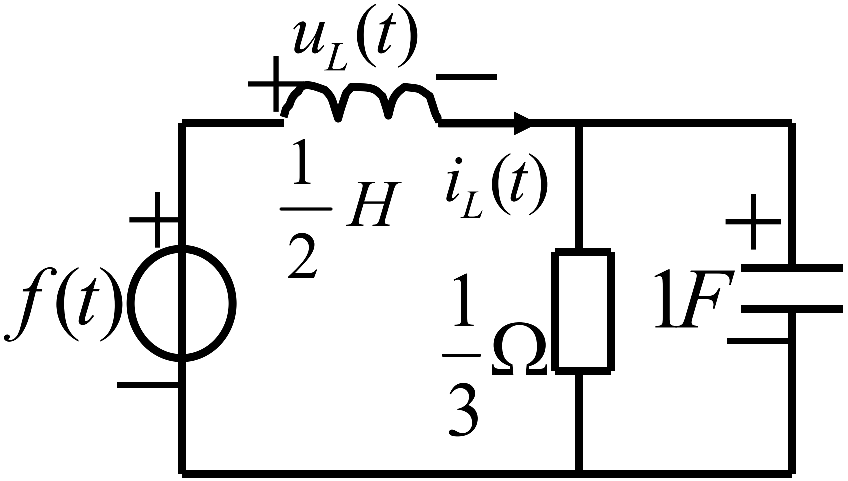
四. 关于初始值的说明

五. 系统的全响应

六. 连续LTI系统时域分析

一. 微分方程的编写

例



已知：

$$u_c(0\_)=2V,$$
$$i_L(0\_)=1A,$$
$$f(t)=1V,t>0$$
$$y_1(t)=u_C(t)$$
$$y_2(t)=i_L(t)$$
$$y_3(t)=u_L(t),$$

求：列出分别以 $y_1(t), y_2(t), y_3(t)$ 为输出的微分方程

解：

$$y_1''(t)+3y_1'(t)+2y_1(t)=2f(t) \quad (1)$$

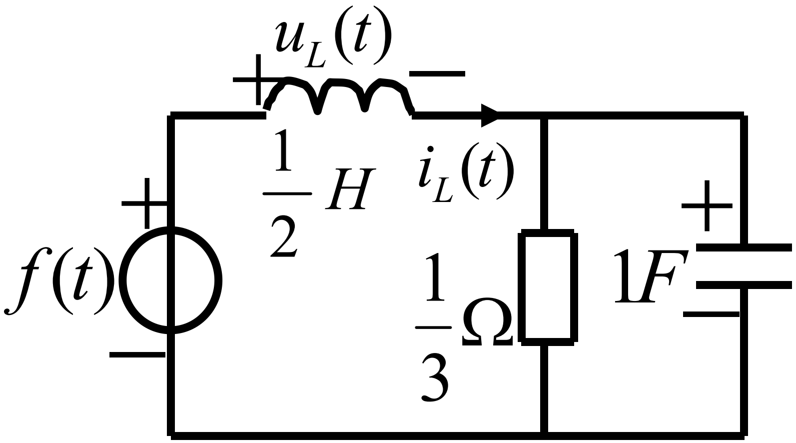
$$y_2''(t)+3y_2'(t)+2y_2(t)=6f(t)+2f'(t) \quad (2)$$

$$y_3''(t)+3y_3'(t)+2y_3(t)=f''(t)+3f'(t) \quad (3)$$

二. 零输入响应 $y_{zi}(t)$

1.定义： 系统的输入为零，完全由系统的初始状态引起的响应.

例



已知：
$u_c(0\_) = 2V,$
$i_L(0\_) = 1A,$
$f(t) = 0, t \geq 0$
$y(t) = i_L(t)$

$$y''(t) + 3y'(t) + 2y(t) = 6f(t) + 2f'(t) \quad (1)$$

$$y''_{zi}(t) + 3y'_{zi}(t) + 2y_{zi}(t) = 0 \quad (2)$$

2.求解： $y(0\_) = y_{zi}(0) = 1, y'(0\_) = y'_{zi}(0) = -4 \quad (3)$

$$y_{zi}(t) = C_1 e^{-2t} + C_2 e^{-t} \quad (4) \qquad \Rightarrow C_1 = 3, C_2 = -2$$

$$y_{zi}(t) = 3e^{-2t} - 2e^{-t} \quad t \geq 0 \quad (5)$$

例2.1-4　求零输入响应.

$$y(0\_)=1,\ y'(0\_)=5$$

$$y''(t)+5y'(t)+4y(t)=2f'(t)-4f(t)\quad (1)$$

解：

$$y''_{zi}(t)+5y'_{zi}(t)+4y_{zi}(t)=0\quad (2)$$

$$\lambda^2+5\lambda+4=0\quad (3)\qquad \lambda_1=-1\quad \lambda_2=-4$$

$$y_{zi}(t)=C_1e^{-t}+C_2e^{-4t}\quad (4)$$

$$\begin{cases} C_1+C_2=1 \\ -C_1-4C_2=5 \end{cases}\quad (5)$$

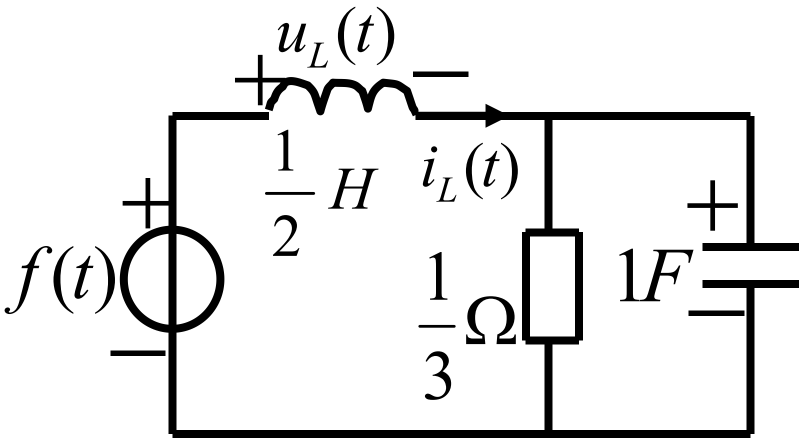
$$\begin{cases} C_1=3 \\ C_2=-2 \end{cases}\quad (6)$$

$$y_{zi}(t)=3e^{-t}-2e^{-4t},\quad t\geq 0\quad (7)$$

三. 零状态响应 $y_{zs}(t)$

1.定义:系统的初始状态为零,完全由系统的输入引起的响应.

例



已知: $u_c(0\_)=0V,$

$i_L(0\_)=0A,$

$f(t)=1V,t\geq 0$

$y(t)=i_L(t)$

$$y''(t)+3y'(t)+2y(t)=6f(t)+2f'(t)$$

2.证明:

$$f(t) \longrightarrow \boxed{\{x(0)=0\}} \longrightarrow y_{zs}(t)$$

$$\delta(t) \xrightarrow{\qquad\qquad} h(t)$$

$$\delta(t-\tau) \xrightarrow{\qquad\qquad} h(t-\tau)$$

$$f(\tau)\delta(t-\tau) \xrightarrow{\qquad\qquad} f(\tau)h(t-\tau)$$

$$\int_{-\infty}^{+\infty} f(\tau)\delta(t-\tau)d\tau \xrightarrow{\qquad\qquad} \int_{-\infty}^{+\infty} f(\tau)h(t-\tau)d\tau$$

$$f(t)*\delta(t) \xrightarrow{\qquad\qquad} f(t)*h(t)$$

$$f(t) \xrightarrow{\qquad\qquad} y_{zs}(t)$$

$$y_{zs}(t) = f(t)*h(t)$$

例 已知系统:

$$y''(t)+3y'(t)+2y(t)=f(t)$$ 求: $y_{zs}(t)$

$$f(t)=\varepsilon(t)$$

$$h''(t)+3h'(t)+2h(t)=\delta(t)$$

$$h(t)=\left[e^{-t}-e^{-2t}\right]\varepsilon(t)$$

$$y_{zs}(t)=\left[e^{-t}-e^{-2t}\right]\varepsilon(t)*\varepsilon(t)$$

$$y_{zs}(t)=\left[\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}\right]\varepsilon(t)$$

例 2.1-5 已知： $y''(t)+5y'(t)+4y(t)=2f'(t)-4f(t)$

$f(t)=\varepsilon(t)$ **求：** $y_{zs}(t)$

$$h''(t)+5h'(t)+4h(t)=2\delta'(t)-4\delta(t)$$

$$h(t)=\left[C_1e^{-t}-C_2e^{-4t}\right]\varepsilon(t)$$

$$h'(t)=\left[C_1+C_2\right]\delta(t)+\left[-C_1e^{-t}-4C_2e^{-4t}\right]\varepsilon(t)$$

$$h''(t)=\left[C_1+C_2\right]\delta'(t)+\left[-C_1-4C_2\right]\delta(t)+\left[C_1e^{-t}+16C_2e^{-4t}\right]\varepsilon(t)$$

$$\begin{cases} C_1+C_2=2 \\ (-C_1-4C_2)+5(C_1+C_2)=-4 \end{cases} \qquad \begin{cases} C_1=-2 \\ C_2=4 \end{cases}$$

$$h(t)=\left[-2e^{-t}+4e^{-4t}\right]\varepsilon(t)$$

$$y_{zs}(t)=f(t)*h(t)=\left[-2e^{-t}+4e^{-4t}\right]\varepsilon(t)*\varepsilon(t)$$

$$=\left[2e^{-t}-e^{-4t}-1\right]\varepsilon(t)$$

四. 关于初始值的说明

1. $y(0_-)$:不考虑输入信号的作用,仅由初始状态产生的输出响应在 $t=0$ 时刻的值, 即零输入响应的初始值:

$$y(0_-)=y_{zi}(0)=y_x(0)$$

2. $y(0_+)$:考虑输入信号及初始状态的共同作用下,输出响应在 $t=0$ 时刻的值,即全响应的初始值:

$$y(0_+)=y_{zi}(0)+y_{zs}(0)$$

五. 全响应

$$y(t)=y_{zi}(t)+y_{zs}(t)=y_x(t)+y_f(t)$$

$$\begin{cases} y''_{zi}(t)+a_1y'_{zi}(t)+a_0y_{zi}(t)=0 \\ y_{zi}(0)=y(0_-) \quad y'_{zi}(0)=y(0_-) \end{cases}$$

$$y_{zs}(t)=y_f(t)=f(t)*h(t)$$

$$h(t)=\left[C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}\right]\varepsilon(t)$$ 比较系数法确定 C_1,C_2

例 已知系统 $y''(t)+4y'(t)+3y(t)=f(t)$ $[ex.2.4(1)]$

求全响应 $f(t)=\varepsilon(t), y'(0_-)=y(0_-)=1$

(1). $y_{zi}(t)$ $$y''_{zi}(t)+4y'_{zi}(t)+3y_{zi}(t)=0$$

$$y_{zi}(t)=A_1e^{-t}+A_2e^{-3t} \qquad y'_{zi}(0)=y_{zi}(0)=1$$

$$y_{zi}(t)=2e^{-t}-e^{-3t}\,,\ t\geq 0$$

(2). $y_{zs}(t)$ $$y_{zs}(t)=f(t)*h(t)$$

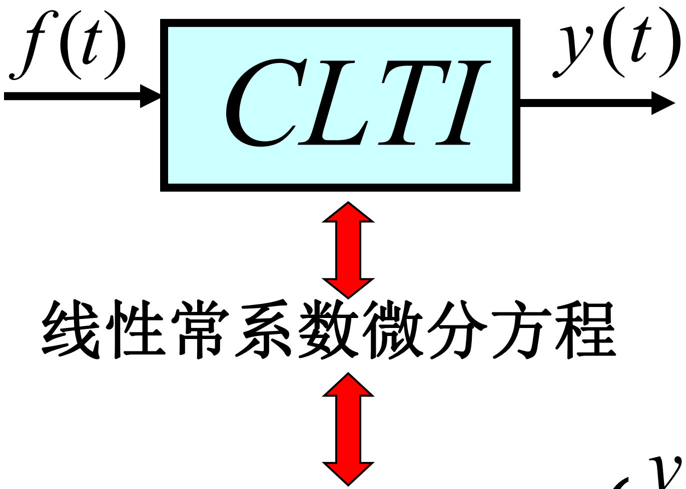
$$h''(t)+4h'(t)+3h(t)=\delta(t)$$

$$h(t)=\left[C_1e^{-t}+C_2e^{-3t}\right]\varepsilon(t)=0.5\left[e^{-t}-e^{-3t}\right]\varepsilon(t)$$

$$y_{zs}(t)=0.5\left[e^{-t}-e^{-3t}\right]\varepsilon(t)*\varepsilon(t)\ =\left[\frac{1}{3}-\frac{1}{2}e^{-t}+\frac{1}{6}e^{-3t}\right]\varepsilon(t)$$

(3). $y(t)$ $$y(t)=y_{zi}(t)+y_{zs}(t)\ =\frac{1}{3}+\frac{3}{2}e^{-t}-\frac{5}{6}e^{-3t}\,, t\geq 0$$

六. 连续LTI系统时域分析



线性常系数微分方程

模拟框图

$$\begin{cases}\text{经典解法}\quad y_h(t)+y_p(t)\\ \text{时域解法}\quad y_{zs}(t)+y_{zi}(t)\end{cases}$$

$$\begin{cases}y_{zi}(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}\\ C_1,C_2\ \text{由与}\ f(t)\ \text{无关的初始值决定}\end{cases}$$

(卷积运算)

$$y_{zi}(0)=y(0\_),\ y_{zi}'(0)=y'(0\_)$$

$$\begin{cases}y_{zs}(t)=y_f(t)=f(t)*h(t)\\ h(t)\ \text{的求取}\qquad \text{匹配系数法}\end{cases}$$

第三章　离散系统的时域分析

要点:

1. $\delta(k), \varepsilon(k)$

2. $h(k), g(k)$

3. 卷积和　$f_1(k) * f_2(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i)$

4. 离散系统时域分析　$y(k) = y_{zi}(k) + y_{zs}(k)$

$$= y_x(k) + y_f(k)$$

3.1 离散信号与系统概述（§1.1 §3.2）

一. 时域分析

1.

线性常系数微分方程

$$\begin{cases} \text{经典解法} & y_h(t)+y_p(t) \\ \text{时域解法} & y_{zs}(t)+y_{zi}(t) \end{cases}$$

模拟框图

$$\begin{cases} y_{zi}(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t} \\ C_1,C_2 \text{ 由与 } f(t) \text{ 无关的初始值决定} \end{cases}$$

(卷积运算)

$$\begin{cases} y_{zs}(t)=y_f(t)=f(t)*h(t) \\ h(t) \text{ 的求取} \end{cases}$$

“匹配系数法”

2.

线性常系数差分方程

$$\begin{cases} \text{经典解法} & y_h(k)+y_p(k) \\ \text{时域解法} & y_{zs}(k)+y_{zi}(k) \end{cases}$$

模拟框图

$$\begin{cases} y_{zs}(k)=C_1\lambda_1^k+C_2\lambda_2^k \\ C_1,C_2\text{ 由与 } f(k)\text{ 无关的初始值决定} \end{cases}$$

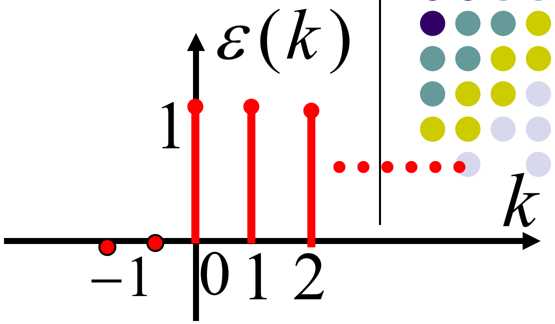
(卷积和运算)

$$\begin{cases} y_{zs}(k)=f(k)*h(k) \\ h(k)\text{ 的求取} \quad \text{“迭代法求初值”} \end{cases}$$

二. 基本离散信号(序列)

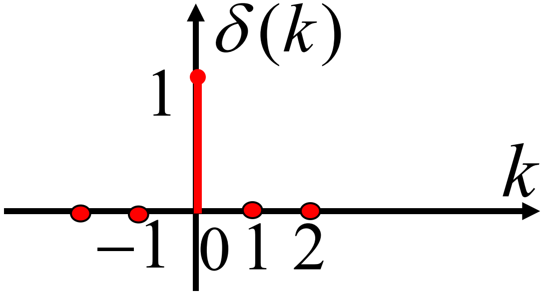
1. 单位阶跃序列

$$\varepsilon(k)=\begin{cases}1,\ldots\ldots \underline{k\geq 0} \\ 0,\ldots\ldots k<0\end{cases}$$



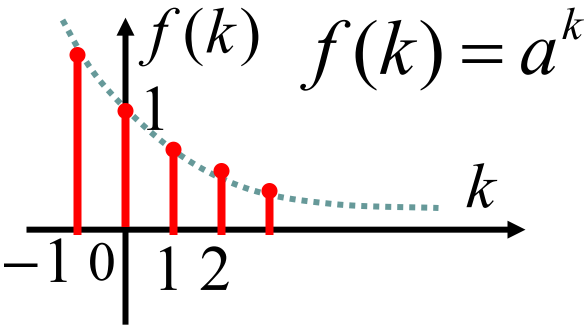
2. 单位(冲激)序列

$$\delta(k)=\begin{cases}1,\ldots\ldots k=0 \\ 0,\ldots\ldots k\neq 0\end{cases}$$

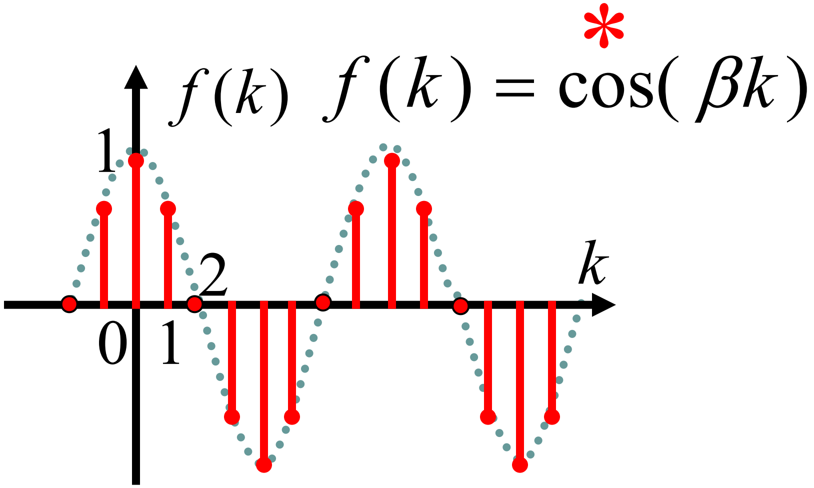


3. 指数序列 $f(k)=a^k$

4. 正弦序列 $f(k)=\cos(\beta k)$



$f(k)=a^k$

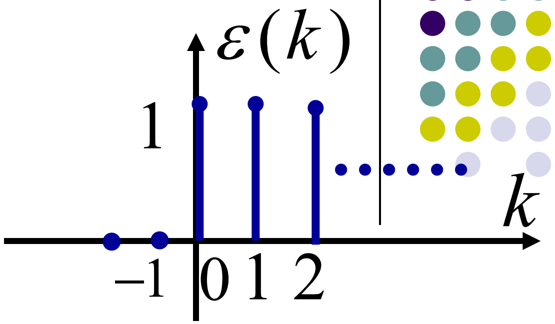


$f(k)=\cos(\beta k)$

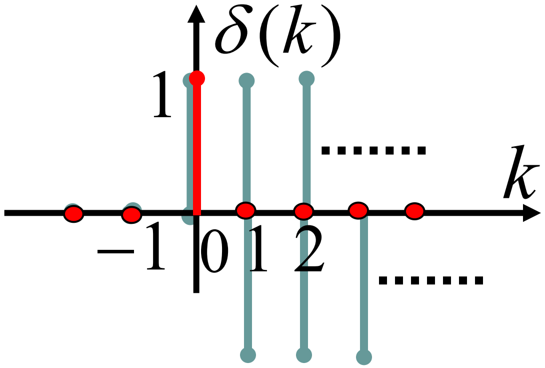
三. 单位序列的性质

1. $\delta(k)=\delta(-k)$

2. $\delta(k)=\nabla\varepsilon(k)=\varepsilon(k)-\varepsilon(k-1)$



3. $\varepsilon(k)=\sum_{j=-\infty}^{k}\delta(j)$



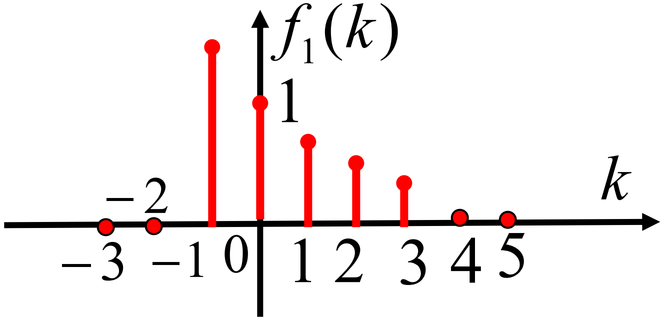
4. $f(k).\delta(k-N)=f(N).\delta(k-N)$

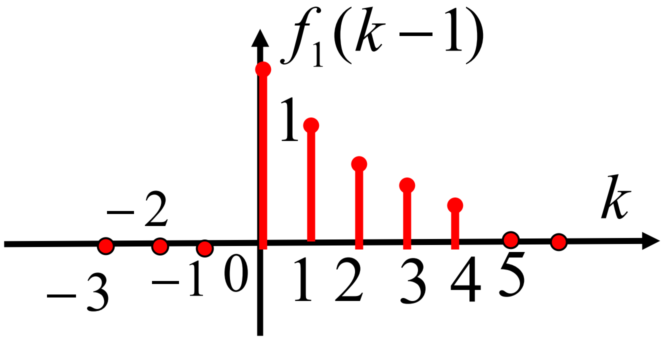
5. $\sum_{k=-\infty}^{\infty}f(k).\delta(k-N)=f(N)$

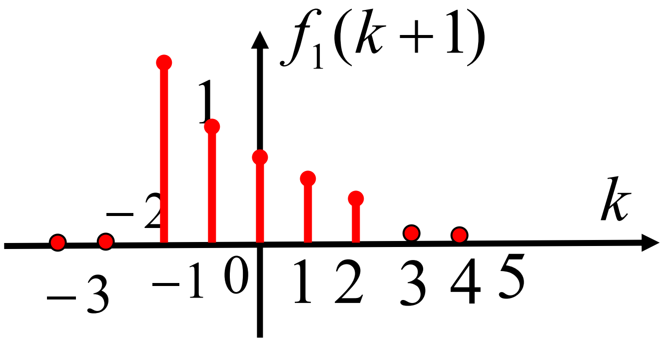
6. $f(k)*\delta(k)=f(k)$ $\quad f(k)*\delta(k-N)=f(k-N)$

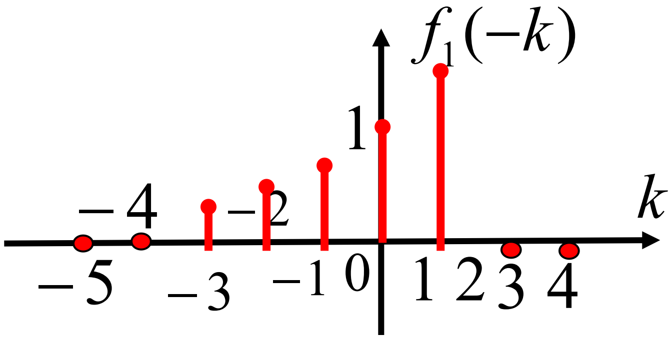
四. 序列的基本运算

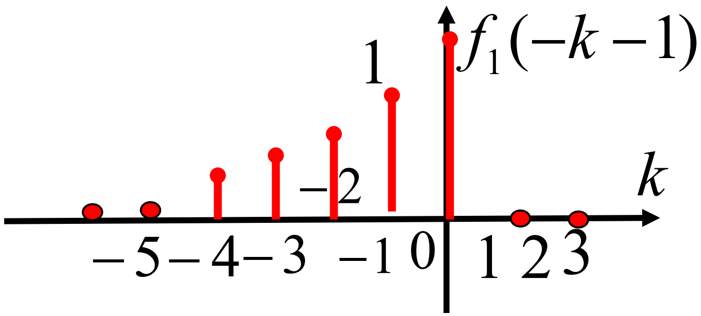
1. 反折和平移:

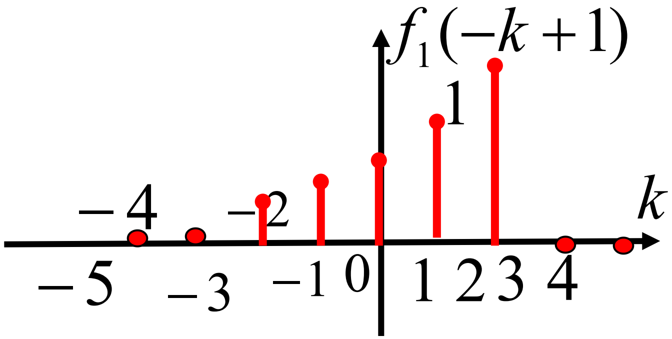




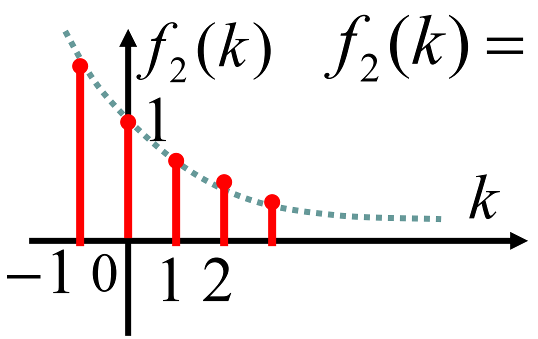






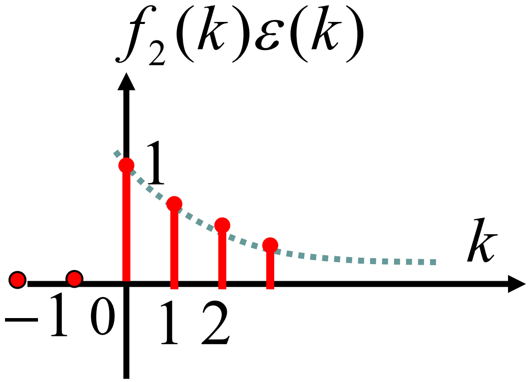


2. 基本函数的使用

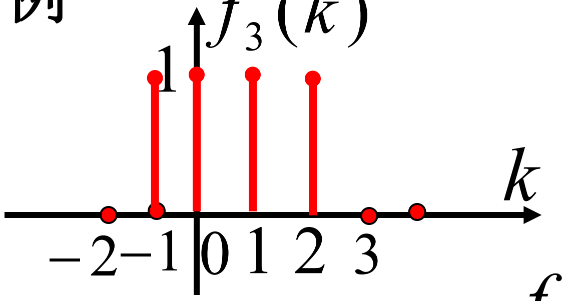


$$f_2(k)=a^k$$

例



例



$$f_3(k)=\varepsilon(k+1)-\varepsilon(k-3)$$

$$f_3(k)=\delta(k+1)+\delta(k)+\delta(k-1)+\delta(k-2)$$

五. 离散系统描述—差分方程

1. 序列的差分

$$\nabla f(k)=f(k)-f(k-1) \qquad \Delta f(k)=f(k+1)-f(k)$$

例. 求：$\nabla f(k)$, $ex.3.1(p.110)$

$$f(k)=\begin{cases}0, k<0\\(\frac{1}{2})^k, k\geq 0\end{cases}$$

$$\nabla f(k)=f(k)-f(k-1)$$

$$=\begin{cases}0, \quad k<0\\1, \quad k=0\\(\frac{1}{2})^k-(\frac{1}{2})^{k-1}=-(\frac{1}{2})^k, \quad k\geq 1\end{cases}$$

$$\nabla f(0)=f(0)-f(-1)$$

2. 差分方程

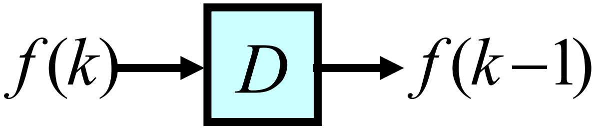
例.ex.3.23(P.113)贷款 M万元,月利率 $\beta=1\%$,定期月初还款$f(k)$万元,贷款余额 $y(k)$ 万元,建立差分方程.

$$y(k)=(1+\beta)y(k-1)-f(k)$$

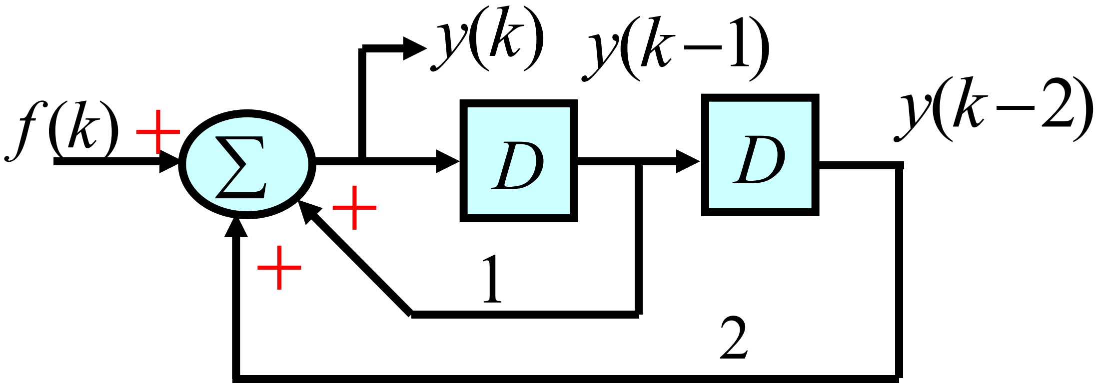
$$y(k)-(1+\beta)y(k-1)=-f(k)$$

3. 离散系统的框图描述

单位延迟器



例 3.2-1 求: 差分方程



解:

$$y(k)=y(k-1)+2y(k-2)+f(k) \qquad y(k)-y(k-1)-2y(k-2)=f(k)$$

4. 离散LTI系统的主要性质

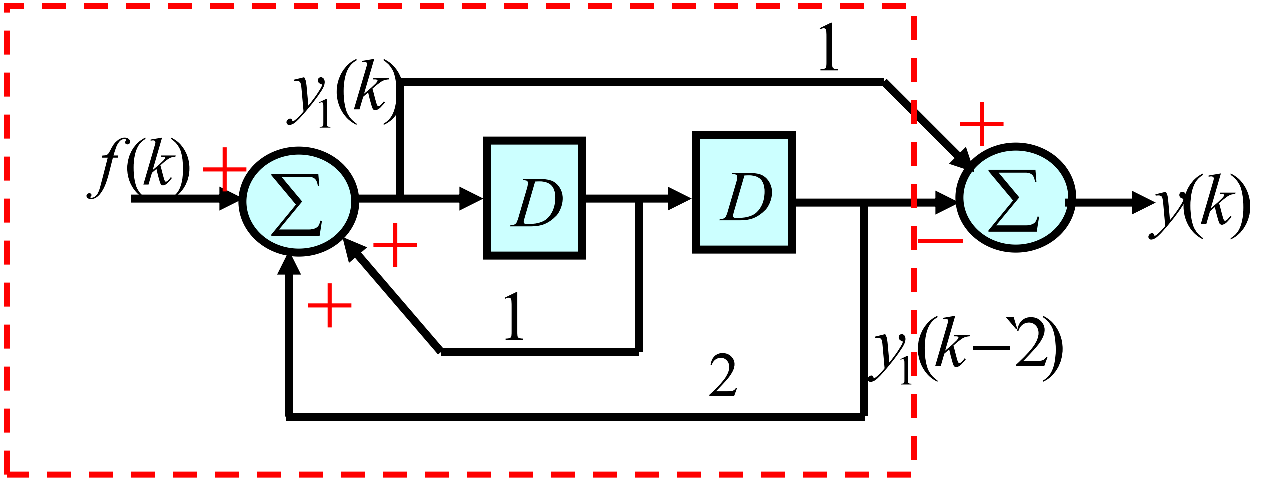
$$f(k)\to y(k)$$

(1) 线性: $a_1 f_1(k)+a_2 f_2(k)\to a_1 y_1(k)+a_2 y_2(k)$

(2) 时移不变性: $f(k-N)\to y(k-N)$

例 3.2-2 求:

差分方程



解:

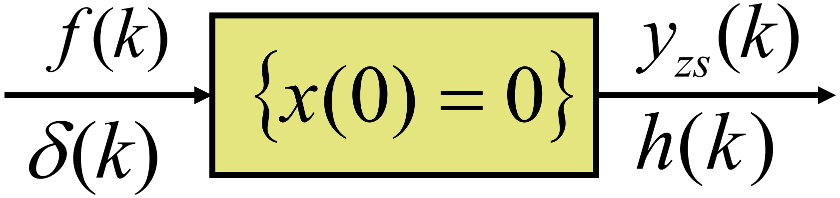
$$y_1(k)-y_1(k-1)-2y_1(k-2)=f(k)\qquad y(k)=y_1(k)-y_1(k-2)$$

$$y(k)-y(k-1)-2y(k-2)=f(k)-f(k-2)$$

3.2 单位序列响应和单位阶跃响应（§3.2）

一. 单位序列响应

1. 定义



$$f(k) \rightarrow \boxed{\{x(0)=0\}} \rightarrow y_{zs}(k)$$

$$\delta(k) \rightarrow \boxed{\{x(0)=0\}} \rightarrow h(k)$$

2. $h(k)$ 的形式

$$y(k)+a_1y(k-1)+a_0y(k-2)=f(k)$$

$$h(k)+a_1h(k-1)+a_0h(k-2)=\delta(k)$$

$$\Rightarrow \lambda^2+a_1\lambda+a_0=0 \Rightarrow \lambda_1,\lambda_2$$

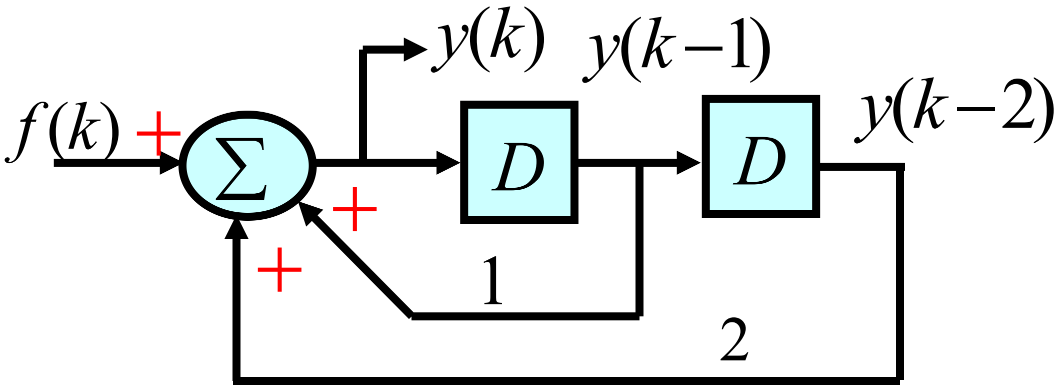
$$\begin{cases} h(k)=\left(C_1\lambda_1^k+C_2\lambda_2^k\right)\underline{\varepsilon(k)} \quad * \\ h(0),h(1) \end{cases}$$

3. *h(k)* 的求取

迭代法求初值

例 3.2-1 求: *h(k)*

解:



$$y(k)-y(k-1)-2y(k-2)=f(k)$$

$$h(k)-h(k-1)-2h(k-2)=\delta(k)$$

$$h(0)=h(-1)+2h(-2)+\delta(k)=1$$

$$h(1)=h(0)+2h(-1)+\delta(1)=1$$

$$\lambda^2-\lambda-2=0 \Rightarrow \lambda_1=-1, \lambda_2=2$$

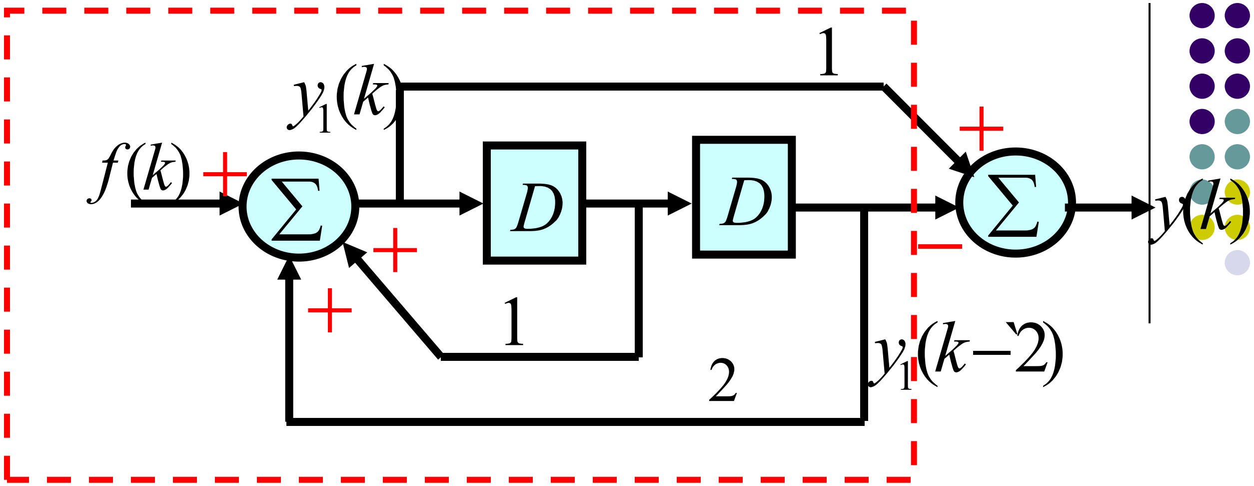
$$h(k)=\left[C_1(-1)^k+C_2(2)^k\right]\varepsilon(k)$$

$$\begin{cases} C_1+C_2=1 \\ C_1(-1)+C_2(2)=1 \end{cases}$$

$$C_1=\frac{1}{3}, C_2=\frac{2}{3}$$

$$h(k)=\left[\frac{1}{3}(-1)^k+\frac{2}{3}(2)^k\right]\varepsilon(k)$$

例 3.2-2 求: ***h(k)***



解:

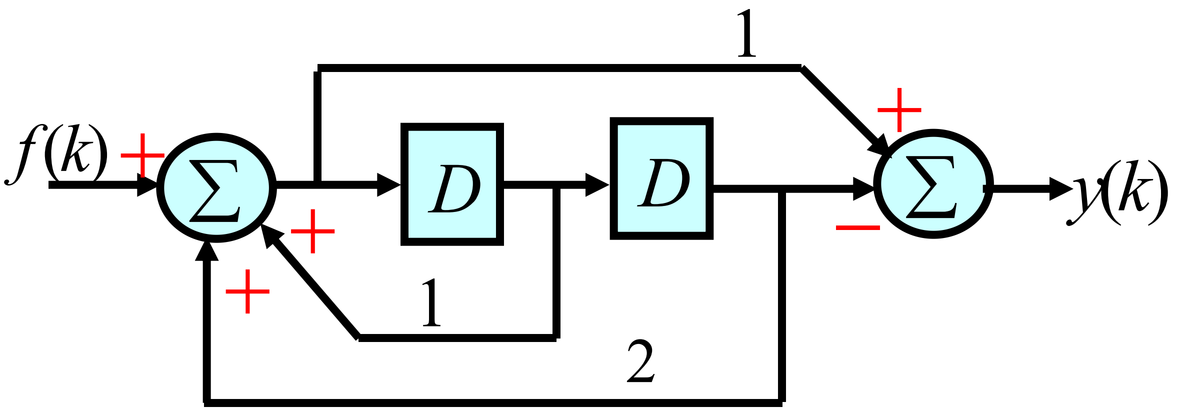
$$y_1(k)-y_1(k-1)-2y_1(k-2)=f(k)$$

$$h_1(k)=\left[\frac{1}{3}(-1)^k+\frac{2}{3}(2)^k\right]\varepsilon(k)$$

$$y(k)-y(k-1)-2y(k-2)=f(k)-f(k-2)$$

$$h(k)=h_1(k)-h(k-2)$$

$$h(k)=\left[\frac{1}{3}(-1)^{\underline{k}}+\frac{2}{3}(2)^{\underline{k}}\right]\underline{\varepsilon(k)}-\left[\frac{1}{3}(-1)^{\underline{k-2}}+\frac{2}{3}(2)^{\underline{k-2}}\right]\underline{\varepsilon(k-2)}$$

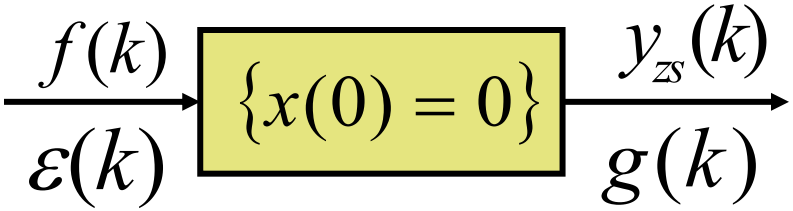


$$h(k)=\left[\frac{1}{3}(-1)^{\underline{k}}+\frac{2}{3}(2)^{\underline{k}}\right]\underline{\varepsilon(k)}-\left[\frac{1}{3}(-1)^{\underline{k-2}}+\frac{2}{3}(2)^{\underline{k-2}}\right]\underline{\varepsilon(k-2)}$$

$$=\begin{cases}0, & k<0\\ \frac{1}{3}(-1)^{k}+\frac{2}{3}(2)^{k}, & k=0\,,1\\ \frac{1}{2}(2)^{k}, & k\geq 2\end{cases}$$

二. 单位阶跃响应

1. 定义



$f(k)$, $\varepsilon(k)$

$\{x(0)=0\}$

$y_{zs}(k)$, $g(k)$

2. *g(k)* 的求取

$$\varepsilon(k)=\sum_{i=-\infty}^{k}\delta(i) \qquad \Rightarrow g(k)=\sum_{i=-\infty}^{k}h(i)$$

例3.2-3 求*g(k)*

$$y(k)-y(k-1)-2y(k-2)=f(k) \qquad (1)$$

$$h(k)=\left[\frac{1}{3}(-1)^k+\frac{2}{3}(2)^k\right]\varepsilon(k)$$

$$g(k)=\left[\frac{1}{3}\sum_{i=0}^{k}(-1)^i+\frac{2}{3}\sum_{i=0}^{k}(2)^i\right].\varepsilon(k)$$

$$=\left[\frac{1}{6}(-1)^k+\frac{4}{3}(2)^k-\frac{1}{2}\right]\varepsilon(k)$$

3.3 卷积和（§3.3）

一. 序列的卷积和

定义： $f_1(k), f_2(k), -\infty < k < \infty$

$$f_1(k) * f_2(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i)$$

例3.3-1(2) $f_1(k) = (0.5)^k \varepsilon(k), f_3(k) = \varepsilon(k)$

计算： $y(k) = f_1(k) * f_3(k)$

$$y(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i) = \sum_{i=-\infty}^{\infty} (0.5)^i \varepsilon(i) \varepsilon(k-i)$$

$$= \left[\sum_{i=0}^{k} (0.5)^i\right] \varepsilon(k) = \frac{1-(0.5)^{k+1}}{1-0.5} \varepsilon(k) = 2\left[1-(0.5)^{k+1}\right] \varepsilon(k)$$

二. 卷积和的性质

(1) $$f_1(k) * f_2(k) = f_2(k) * f_1(k)$$

(2) $$f_1(k) * \left[f_2(k) + f_3(k) \right] = f_1(k) * f_2(k) + f_1(k) * f_3(k)$$

(3) $$f_1(k) * \left[f_2(k) * f_3(k) \right] = \left[f_1(k) * f_2(k) \right] * f_3(k)$$

(4) $$f(k) * \delta(k - N) = f(k - N)$$ *

$$a\delta(k-N_1) * b\delta(k-N_2) = (a \times b)\delta(k-N_1-N_2)$$ *

$$x_1(k-N_1) * x_2(k-N_2) = x_1(k) * x_2(k) * \delta(k-N_1-N_2)$$

三. 卷积和的计算

(1) 定义　　例　计算卷积和 $y(k)=\varepsilon(k)*\varepsilon(k)$

解： $$y(k)=\sum_{i=-\infty}^{\infty}\varepsilon(i)\varepsilon(k-i)$$

$$=\left[\sum_{i=0}^{k}1\right].\varepsilon(k)=(k+1).\varepsilon(k)$$

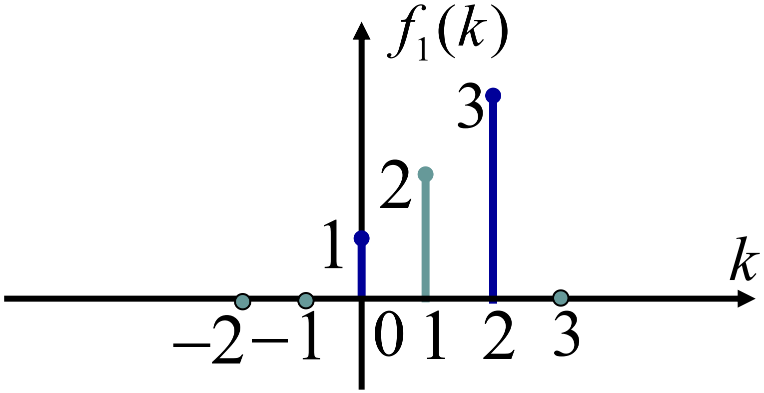
例　计算卷积和 $y(k)=\left[a^k\varepsilon(k)\right]*\left[b^k\varepsilon(k)\right]$

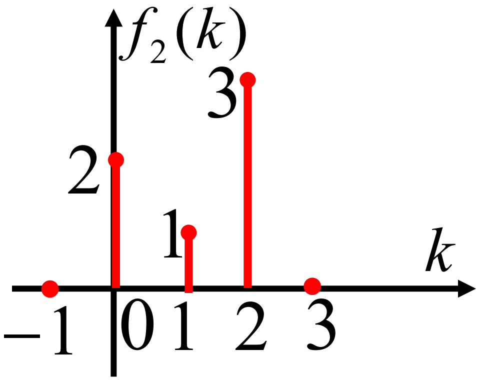
解： $$y(k)=\sum_{i=-\infty}^{\infty}\left[a^k\varepsilon(i)\right]\left[b^{k-i}\varepsilon(k-i)\right]$$

$$=\left[\sum_{i=0}^{k}a^ib^{k-i}\right]\varepsilon(k)=\frac{a^{k+1}-b^{k+1}}{a-b}\varepsilon(k)$$

(2) 图解

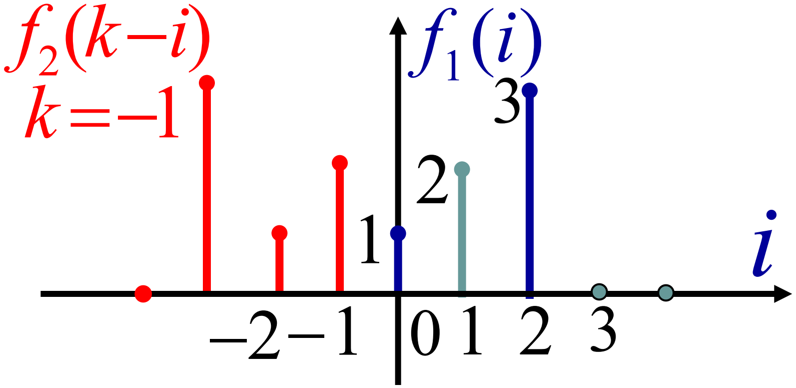
例



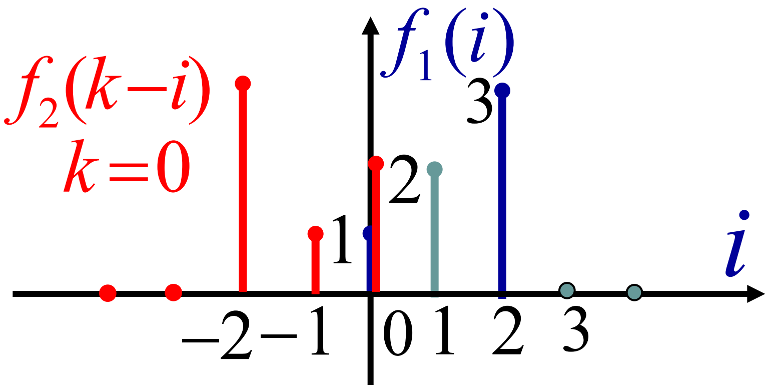


$$\boxed{f_1(k) * f_2(k) = \sum_{i=-\infty}^{\infty} f_1(i) f_2(k-i)}$$

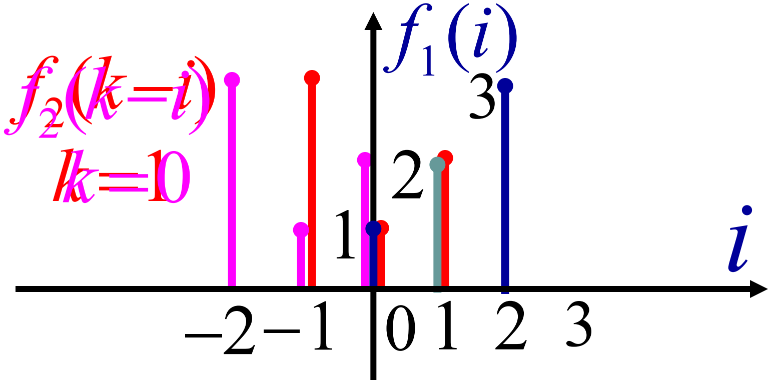
$$y(k) = f_1(k) * f_2(k)$$



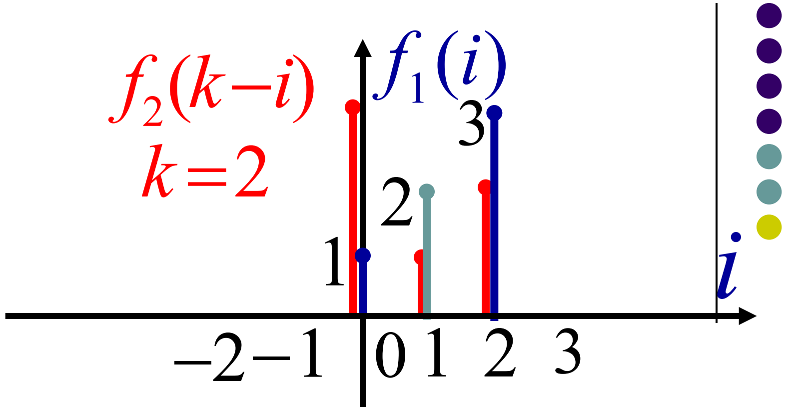
$y(k) = 0, k \leq -1$



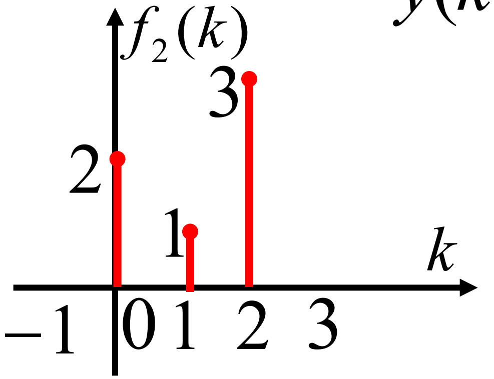
$y(k) = 2, k = 0$

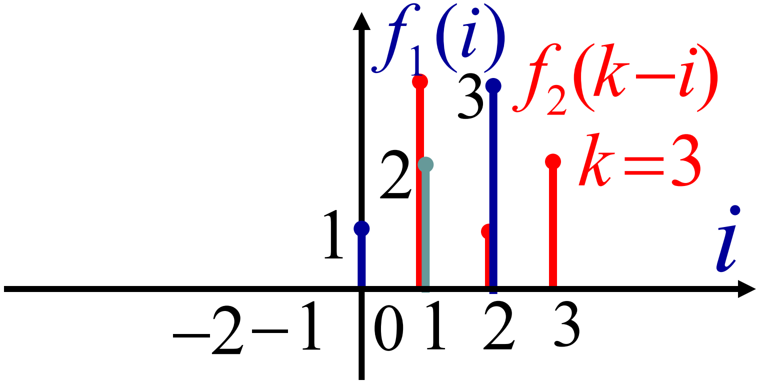


$y(k)=5, k=1$

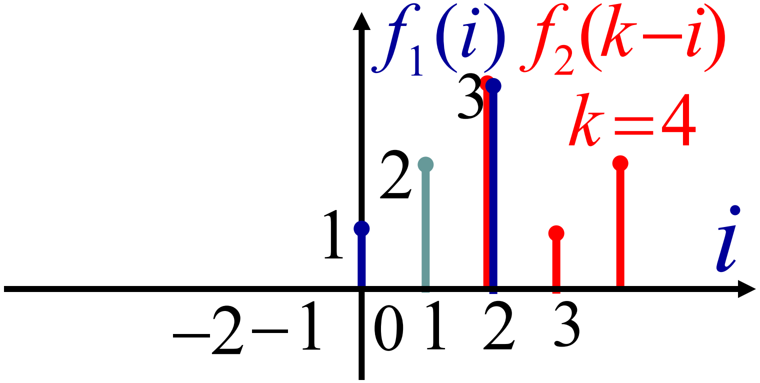


$y(k)=11, k=2$

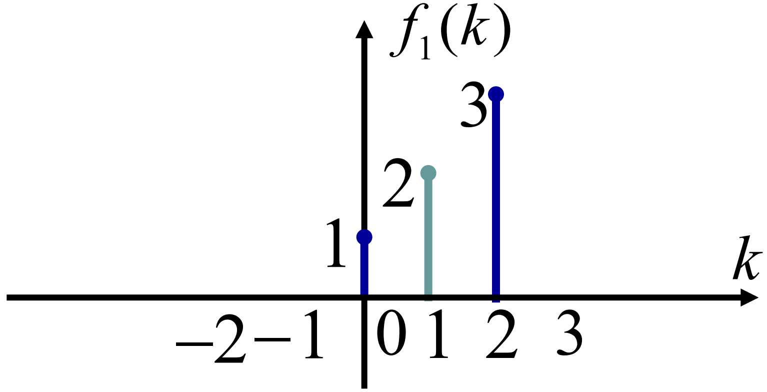


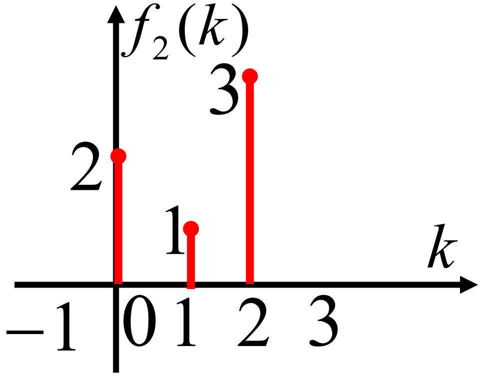


$y(k)=9, k=3$



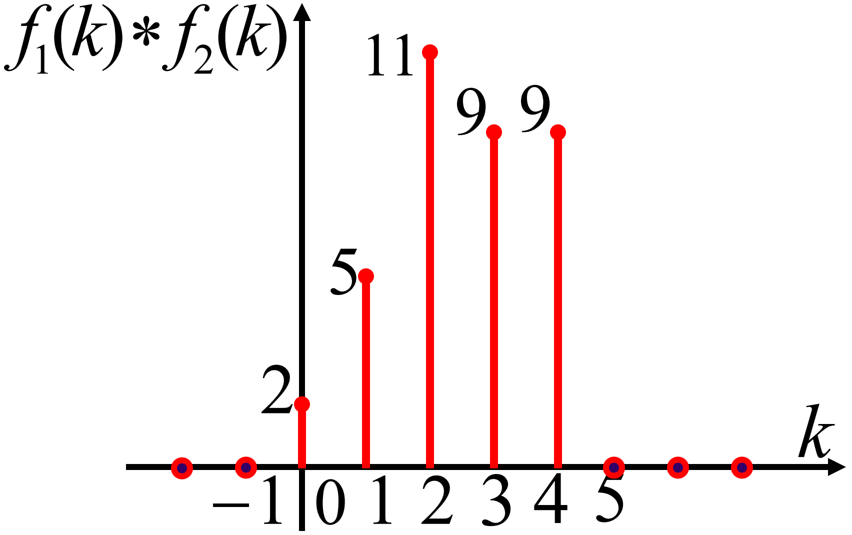
$y(k)=9, k=4$





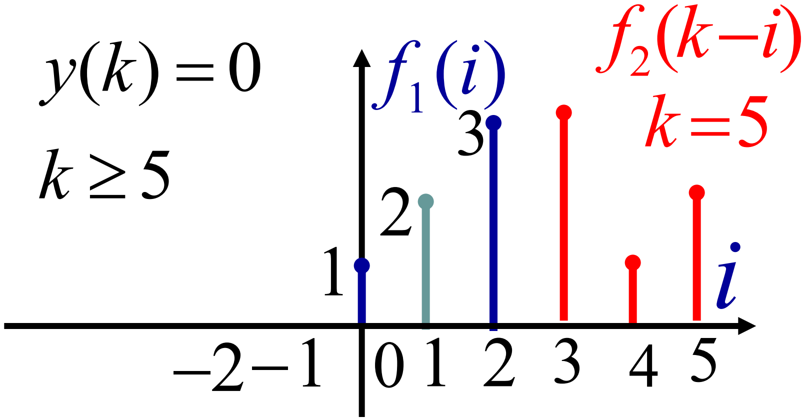
$$f_1(k)=\delta(k)+2\delta(k-1)+3\delta(k-2)$$

$$f_2(k)=2\delta(k)+\delta(k-1)+3\delta(k-2)$$



$y(k)=0$

$k\geq 5$



$$y(k)=2\delta(k)+5\delta(k-1)+11\delta(k-2)+9\delta(k-3)+9\delta(k-4)$$

(3) “矩阵表” $a\delta(k-n)*b\delta(k-m)=(a\times b)\delta(k-n-m)$

例 $f_1(k)=\delta(k)+2\delta(k-1)+3\delta(k-2)$

$f_2(k)=2\delta(k)+\delta(k-1)+3\delta(k-2)$

$$
\begin{aligned}
f_1(k)*f_2(k)= & \\
& 2\times1\delta(k)+2\times2\delta(k-1)+2\times3\delta(k-2)\\
& +1\times1\delta(k-1)+1\times2\delta(k-2)+1\times3\delta(k-3)\\
& +3\times1\delta(k-2)+3\times2\delta(k-3)+3\times3\delta(k-4)\\
\hline
& 2\delta(k)+5\delta(k-1)+11\delta(k-2)+9\delta(k-3)+9\delta(k-4)
\end{aligned}
$$

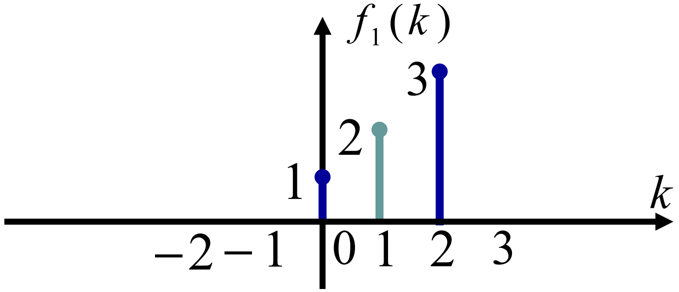
$$
\begin{array}{ccccc}
1 & 2 & 3 & & \\
2 & 1 & 3 & & \\
\hline
2 & 4 & 6 & & \\
 & 1 & 2 & 3 & \\
 & & 3 & 6 & 9 \\
\hline
2 & 5 & 11 & 9 & 9
\end{array}
$$

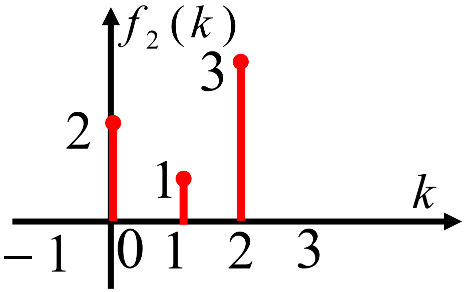
$\leftarrow \boxed{k=2}$

$\leftarrow \boxed{k=2}$

$\uparrow$ $k=0$

$\nwarrow \boxed{k=2+2=4}$





例 计算 $f_1(k)*f_2(k)$

$$f_1(k)=\{\ldots,0,3,0,\underset{\substack{\uparrow\\ k=0}}{2},4,0,\ldots\},\ f_2(k)=\{\ldots,0,-2,\underset{\substack{\uparrow\\ k=0}}{1},5,0,\ldots\}$$

$$y(k)=\{0,-6,3,11,-\underset{\substack{\uparrow\\ k=0}}{6},14,20,0\}$$

例 计算 $y(k)=f_1(k)*f_2(k)$ $y(-1)$,

$$f_1(k)=\{0,1,\underset{\substack{\uparrow\\ k=0}}{2},0,3,0\},\ f_2(k)=\{0,-4,\underset{\substack{\uparrow\\ k=0}}{2},6,0\}$$

$$y(k)=\{0,-4,-6,\underset{\substack{\uparrow\\ k=0}}{10},0,6,18,0\}$$

3.4 离散系统时域分析（§3.1）

↕

线性常系数差分方程

$$\begin{cases}\text{经典解法} & y_h(k)+y_p(k)\\ \text{时域解法} & y_{zs}(k)+y_{zi}(k)\end{cases}$$

↕

模拟框图

$$\begin{cases}y_{zi}(k)=\sum_i C_i\lambda_i^k\\ \text{n个与 } f(k) \text{ 无关的初始值}\end{cases}$$

(卷积和运算)

$$\begin{cases}y_{zs}(k)=f(k)*h(k)\\ h(k)\text{ 的求取} \quad \text{迭代法求初值}\end{cases}$$

一. 零输入响应 $y_{zi}(k)$

$$y_{zi}(k)+a_1 y_{zi}(k-1)+a_0 y_{zi}(k-2)=0$$

$$\Rightarrow \lambda^2 + a_1\lambda + a_0\lambda = 0 \quad \Rightarrow \lambda_1, \lambda_2$$

$$\Rightarrow \begin{cases} y_{zi}(k) = C_1(\lambda_1)^k + C_2(\lambda_2)^k \quad k \geq 0 \\ y_{zi}(0), y_{zi}(1) \end{cases}$$

*

对比： $y''_{zi}(t) + a_1 y'_{zi}(t) + a_0 y_{zi}(t) = 0$

$$\begin{cases} y_{zi}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ y_{zi}(0), y'_{zi}(0) \end{cases}$$

例 3.1-4 求零输入响应:

$$\begin{cases} y(k)+3y(k-1)+2y(k-2)=f(k);\ f(k)=0,k\geq 0 \\ y(-1)=0,y(-2)=\dfrac{1}{2} \end{cases}$$

解:

$$y(-1)=0,y(-2)=\frac{1}{2}\Rightarrow y_{zi}(0)=-1,y_{zi}(1)=3$$

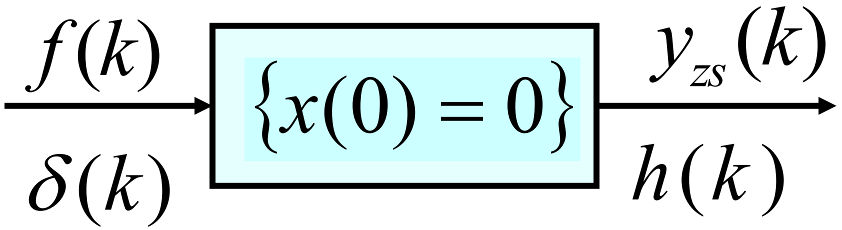
$$\lambda^2+3\lambda+2=0\ \Rightarrow\lambda_1=-1,\lambda_2=-2$$

$$\begin{cases} y_{zi}(k)=C_1(-1)^k+C_2(-2)^k \\ y(-1)=y_{zi}(-1)=0,y(-2)=y_{zi}(-2)=\dfrac{1}{2} \end{cases}$$

$$\Rightarrow\begin{cases} C_1+C_2=-1 \\ -C_1-2C_2=3 \end{cases}\Rightarrow\begin{cases} C_1=1 \\ C_2=-2 \end{cases}$$

$$y_{zi}(k)=(-1)^k-2(-2)^k,k\geq 0$$

二. 零状态响应 $y_{zs}(k)$



$$\delta(k) \longrightarrow h(k)$$

$$\delta(k-i) \longrightarrow h(k-i)$$

$$f(i)\delta(k-i) \longrightarrow f(i)h(k-i)$$

$$\sum_{i=-\infty}^{\infty} f(i)\delta(k-i) \longrightarrow \sum_{i=-\infty}^{\infty} f(i)h(k-i)$$

$$f(k) \longrightarrow y_{zs}(k) = \sum_{i=-\infty}^{\infty} f(i)h(k-i) = f(k)*h(k)$$

例　求 $y_{zs}(k)$ 已知：$h(k)=(0.5)^k\varepsilon(k),$

$$f(k)=\varepsilon(k)-\varepsilon(k-5)$$

解：$y_{zs}(k)=h(k)*f(k)$

$$=(0.5)^k\varepsilon(k)*\left[\varepsilon(k)-\varepsilon(k-5)\right]$$

$$(0.5)^k\varepsilon(k)*\varepsilon(k)=[\sum_{i=0}^{k}(0.5)^i].\varepsilon(k)$$

$$=2[1-(0.5)^{k+1}]\varepsilon(k)$$

$$y_{zs}(k)=2\left[1-(0.5)^{k+1}\right]\varepsilon(k)-2\left[1-(0.5)^{k-4}\right]\varepsilon(k-5)$$

$$=\begin{cases}0,\ldots\ldots\ldots\ldots\ldots\ldots\ldots k<0\\ 2-(0.5)^k,\ldots\ldots\ldots 0\le k\le 4\\ 31(0.5)^k,\ldots\ldots\ldots\ldots k\ge 5\end{cases}$$

例 3.1-5 求 $y_{zs}(k)$已知:

$$y(k)+3y(k-1)+2y(k-2)=f(k),\quad f(k)=2^k\varepsilon(k)$$

解: $h(k)+3h(k-1)+2h(k-2)=\delta(k)$

$$h(k)=[C_1(-1)^k+C_2(-2)^k]\varepsilon(k)$$

$$h(0)=1\qquad h(1)=-3$$

$$h(k)=[-(-1)^k+2(-2)^k]\varepsilon(k)$$

$$a^k\varepsilon(k)*b^k\varepsilon(k)$$

$$=\frac{a^{k+1}-b^{k+1}}{a-b}\varepsilon(k)$$

$$y_{zs}(k)=f(k)*h(k)$$

$$=[-(-1)^k+2(-2)^k]\varepsilon(k)*2^k\varepsilon(k)$$

$$=\left[-\frac{1}{3}(-1)^k+(-2)^k+\frac{1}{3}(2)^k\right]\varepsilon(k)$$

三. 全响应 $y(k)=y_{zs}(k)+y_{zi}(k)$

例 求全响应 $y(k), k\geq 0$,已知:

$$\begin{cases} y(k)-0.7y(k-1)+0.12y(k-2)=2f(k)-f(k-1) \\ y_{zi}(0)=8, y_{zi}(1)=3, f(k)=(0.2)^k\varepsilon(k) \end{cases}$$

解: **(1)** $\lambda^2-0.7\lambda+0.12=0 \to \lambda_1=0.3, \lambda_2=0.4$

$$\begin{cases} y_{zi}(k)=C_1(0.3)^k+C_2(0.4)^k \\ y_{zi}(0)=8, y_{zi}(1)=3 \end{cases} \Rightarrow \begin{cases} C_1=2 \\ C_2=6 \end{cases}$$

$$y_{zi}(k)=2(0.3)^k+6(0.4)^k \quad k\geq 0$$

(2) $h_1(k)-0.7h_1(k-1)+0.12h_1(k-2)=\delta(k)$

$$h_1(k)=\left[A_1(0.3)^k+A_2(0.4)^k\right]\varepsilon(k)$$

$$h_1(0)=1, h_1(1)=0.7$$

$$y(k)-0.7y(k-1)+0.12y(k-2)=f(k)$$

$$h_1(k)=[4(0.4)^k-3(0.3)^k]\varepsilon(k)$$

$$y(k)-0.7y(k-1)+0.12y(k-2)=2f(k)-f(k-1)$$

$$h(k)=2h_1(k)-h_1(k-1)$$

$$=2\left[4(0.4)^{\underline{k}}-3(0.3)^{\underline{k}}\right]\underline{\varepsilon(k)}-\left[4(0.4)^{\underline{k-1}}-3(0.3)^{\underline{k-1}}\right]\underline{\varepsilon(k-1)}$$

$$=2\delta(k)+\left[4(0.3)^k-2(0.4)^k\right]\underline{\varepsilon(k-1)}$$

$$=\left[4(0.3)^k-2(0.4)^k\right]\underline{\varepsilon(k)}$$

(3) $y_{zs}(k)=f(k)*h(k)$ $\quad y_{zi}(k)=2(0.3)^k+6(0.4)^k$

$$=\left[12(0.3)^k-4(0.4)^k-6(0.2)^k\right]\varepsilon(k)$$

(4) $y(k)=y_{zs}(k)+y_{zi}(k)$

$$=2\left[7(0.3)^k+(0.4)^k-0.3(0.2)^k\right],k\geq 0$$